HCF And LCM

Study Notes of HCF and LCM and Aptitude Quiz

Here we are posting some useful notes and concepts for the topic “HCF & LCM” which we hope will be helpful in the Quant section of the exams. The post consists of various important concepts covered under the topic “HCF & LCM”

HCF & LCM are acronym for words, Highest common factor and Lowest common multiple respectively.

1. H. C. F

While we all know what a multiplication is like 2 * 3 = 6. HCF is just the reverse of multiplication which is known as Factorization. Now factorization is breaking a composite number into its prime factors. Like 6 = 2 * 3, where 6 is a composite number and 2 & 3 are prime number.

“In mathematics, the Highest Common Factor (HCF) of two or more integers is the largest positive integer that divides the numbers without a remainder. For example, the HCF of 8 and 12 is 4.”

Calculation

– By Prime Factorizations

Highest Common Factor can be calculated by first determining the prime factors of the two numbers and then comparing those factors, to take out the common factors.

As in the following example: HCF (18, 42), we find the prime factors of 18 = 2 * 3 * 3 and 42 = 7 * 2 * 3 and notice the “common” of the two expressions is 2 * 3; So HCF (18, 42) = 6.

– By Division Method

In this method first divide a higher number by smaller number.

LCM and HCF

- Put the higher number in place of dividend and smaller number in place of divisor.
- Divide and get the remainder then use this remainder as divisor and earlier divisor as dividend.
- Do this until you get a zero remainder. The last divisor is the HCF.
- If there are more than two numbers then we continue this process as we divide the third lowest number by the last divisor obtained in the above steps.

First find H.C.F. of 72 and 126

72|126|1
72
54| 72|1
54
18| 54| 3
54
0

H.C.F. of 72 and 126 = 18

2. L.C.M

The Least Common Multiple of two or more integers is always divisible by all the integers it is derived from. For example, 20 is a multiple of 5 because 5 × 4 = 20, so 20 is divisible by 5 and 2. Because 10 is the smallest positive integer that is divisible by both 5 and 2, it is the least common multiple of 5 and 4.

LCM cam also be understand by this example:

Multiples of 5 are:

5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70 …

And the multiples of 6 are:

LCM and HCF

6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, …

Common multiples of 5 and 6 are:

30, 60, 90, 120, ….

Hence, the lowest common multiple is simply the first number in the common multiple list i.e 30.

Calculation

– By Prime Factorizations

The prime factorization theorem says that every positive integer greater than 1 can be written in only one way as a product of prime numbers.

Example: To find the value of LCM (9, 48, and 21).

First, find the factor of each number and express it as a product of prime number powers.

Like 9 = 32,

48 = 24 * 3

21 = 3 * 7

Then, write all the factors with their highest power like 32, 24, and 7. And multiply them to get their LCM.

Hence, LCM (9, 21, and 48) is 32 * 24 * 7 = 1008.

– By Division Method

Here, divide all the integers by a common number until no two numbers are further divisible. Then multiply the common divisor and the remaining number to get the LCM.

2 | 72, 240, 196
2 | 36, 120, 98
2 | 18, 60, 49
3 | 9, 30, 49
| 3, 10, 49

LCM and HCF

L.C.M. of the given numbers

= product of divisors and the remaining numbers

= 2×2×2×3×3×10×49

= 72×10×49 = 35280.

Relation between L.C.M. and H.C.F. of two natural numbers

The product of L.C.M. and H.C.F. of two natural numbers = the product of the numbers.

For Example:

LCM (8, 28) = 56 & HCF (8, 28) = 4

Now, 8 * 28 = 224 and also, 56 * 4 = 224

TIPS FOR SOLVING QUESTIONS RELATED TO HCF and LCM:

Prime number : A prime number is a natural number greater than 1 that has no positive divisors other than 1 and itself.

For example, 2, 3, 5, 7, 11, 13, etc. are prime numbers.

Co-Prime Number: **Two numbers are said to be relatively prime, mutually prime, or co-prime to each** other when they have no common factor or the only common positive factor of the two numbers is 1.

In other words, two numbers are said to be co-primes if their H.C.F. is 1.

Factors: The numbers are said to be factors of a given number when they exactly divide that number.

Thus, factors of 18 are 1, 2, 3, 6, 9 and 18.

LCM and HCF

Common Factors: A common factor of two or more numbers is a number which divides each of them exactly.

Thus, each of the numbers - 2, 4 and 8 is a common factor of 8 and 24.

Multiple: When a number is exactly divisible by another number, then the former number is called the multiple of the latter number.

Thus, 45 is a multiple of 1, 3, 5, 9, 15 and 45.

Common Multiple: A common multiple of two or more numbers is a number which is exactly divisible by each of them.

For example, 12, 24 and 36 is a common multiple of 3, 4, 6 and 12.

Prime Factorisation: If a natural number is expressed as the product of prime numbers, then the factorisation of the number is called its prime factorisation.

A prime factorisation of a natural number can be expressed in the exponential form.

For example:

(1) 24 = 2 x 2 x 2 x 3 = 2^3 x 3.

(2) 420 = 2 x 2 x 3 x 5 x 7 = 2^2 x 3 x 5 x 7

Highest Common Factor (H.C.F.) or Greatest Common Divisor (G.C.D.) or Greatest Common Measure (G.C.M.) are synonymous terms:

The H.C.F of two or more than two numbers is the greatest numbers which divides each of them without any remainder.

Methods of finding the H.C.F. of a given set of numbers:

Method I: Prime Factorisation method :

Express each one of the given numbers as the product of prime factors. The product of least powers/index of common prime factors gives H.C.F.

LCM and HCF

Example I:

Find the H.C.F. of 8 and 14 by Prime Factorisation method?

Solution:

8 = 2 x 2 x 2

14 = 2 x 7

Common factor of 8 and 14 = 2.

Thus, Highest Common Factor (H.C.F.) of 8 and 14 = 2.

Example II:

Find the H.C.F. of 24, 36 and 72 by Prime Factorisation method?

24 = 2 x 2 x 2 x 3

36 = 2 x 2 x 3 x 3

72 = 2 x 2 x 2 x 3 x 3

H.C.F. of 24, 36 and 72 = Product of common factors with least powers/index = 2^2 x 3

Thus, Highest Common Factor (H.C.F.) of 24, 36 and 72 = 12

Method II: Successive Division method :

Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.

Example I:

Find the H.C.F. of 8 and 14 by Successive Division method?

8 | 14 | 1

8

6 | 8 | 1

LCM and HCF

6

2 | 6 | 3

6

0

Least Common Multiple (L.C.M.):

L.C.M. of two or more given numbers is the smallest number which is divisible by all the given numbers.

Methods of finding the L.C.M. of a given set of numbers:

Method I: Prime Factorisation method :

Express each one of the given numbers as the product of prime factors. The product of greatest powers/index of common prime factors gives L.C.M.

Example I:

Find the L.C.M. of 8 and 14 by Prime Factorisation method?

Solution:

8 = 2 x 2 x 2

14 = 2 x 7

L.C.M. of 8 and 14 = Product of all the prime factors of each of the given number with greatest index of common prime factors

= 2^3 x 7 = 56.

Thus, L.C.M. of 8 and 14 = 56.

Method II: Division method :

Find the L.C.M. of 8 and 14 by using Division method?

2 | 8, 14

|4, 7

L.C.M. of the given numbers = product of divisors and the remaining numbers = 2 x 4 x 7 = 56.

LCM and HCF

Other important formula related to H.C.F. and L.C.M.:

(1) H.C.F. of given fractions = H.C.F. of numerator / L.C.M. of denominator

2) L.C.M. of given fractions = L.C.M. of numerator / H.C.F. of denominator

(3) Product of two numbers (First number x Second Number) = H.C.F. X L.C.M.

(4) H.C.F. of a given number always divides its L.C.M.

(5) Largest number which divides x, y, z to leave remainder R in each case = H.C.F. of (x-R), (y-R), (z-R).

(6) Largest number which divides x, y, z to leave same remainder = H.C.F. of (y-x), (z-y), (z-x).

(7) Largest number which divides x, y, z to leave remainder a,b,c = H.C.F. of (x-a), (y-b), (z-c).

(8) Least number which when divided by x, y, z and leaves a remainder R in each case = (L.C.M. of x, y, z) + R