Projectile motion

Projectile motion

- A projectile is a particle, which is given an initial velocity, and then moves under the action of its weight alone.
  When object moves at constant horizontal velocity and constant vertical downward acceleration, such a two dimension motion is called projectile. The projectile motion can be treated as the resultant motion of two independent component motion taking place simultaneously in mutually perpendicular directions. One component is along the horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to gravitational force.
- Projectile motion is an important example of motion in a plane. Vertical motion of the projectile is the motion along Y axis and horizontal motion is motion along X axis.

Important terms used in projectile motion

When a particle is projected into air, the angle that the direction of projection makes with horizontal plane through the point of projection is called the angle of projection, the path, which the particle describes, is called the trajectory, the distance between the point of projection and the point where the path meets any plane draws through the point of projection is its range, the time that elapses in air is called as time of flight and the maximum distance above the plane during its motion is called as maximum height attained by the projectile

Terms used in describing projectile motion

- Point of projection
- Angle of projection
- Horizontal range
- Time of flight
- Maximum height reached

The motion of projectile can be discussed separately for the horizontal and vertical parts. The origin is taken as the point of projection. The instant the particle is projected is taken as t = 0. X-Y plane is the plane of motion. The horizontal line OX is taken as the X axis. Vertical line OY is the Y axis. Vertically upward direction is taken as positive direction of Y Initial velocity of the particle = uAngle between the velocity and horizontal axis =

ux – x-component of velocity = u cos θax – x component of acceleration = 0
uy – y component of velocity = u sin θ
ay = y component of acceleration = -g

Horizontal motion – Equations of motion

ux = u cos θax = 0vx = ux +axt = ux = u cos θ (as ax = 0)

Hence x component of the velocity remains constant.
Displacement in horizontal direction = x = $u_x t + 1/2 a_x t^2$ As $a_x = 0$, $x = u_x t = ut \cos\theta$ Vertical motion – Equations of motion

$$u_y = u \sin\theta$$
$$a_y = -g$$
$$v_y = u_y - gt$$

Displacement in y direction = $y = u_y t - ½ gt^2$ $v_y^2 = u_y^2 - 2gy$

Time of flight of projectile

A projectile is projected from the ground at point O, and after some travel in the space, it reaches the ground at point B. The time taken for this travel is called time of flight of the projectile. Over this time, the displacement in y direction becomes zero.

Hence we can write $y = u_y t - ½ gt^2 = u \sin\theta * t - ½ gt^2$

solving we get $T = (2u \sin\theta)/g$

Time of flight of the projectile = $(2u \sin\theta)/g$

Range of the projectile:

The distance OB travelled by the projetile in the horizontal direction is the range.OB = $(u^2 \sin 2\theta)/g$
Maximum height reached When the projectile reaches the highest level in vertical direction, the vertical component of the velocity becomes zero. The time taken for the vertical component of velocity to become zero is

$$v_y = 0 = u_y - gt = u \sin\theta - gt$$

So $t = (u \sin\theta)/g$
So time taken for vertical component of velocity to become zero is $(u \sin\theta)/g$.
Note that this time is half of Time of flight.

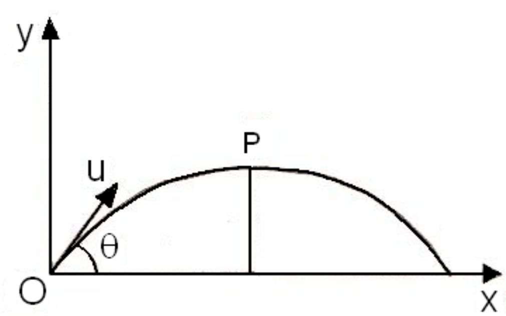
Formula

1. $v_{av} = s/(t_2 - t_1)$
   where vav = average speed
2. $v = \lim_{\Delta t \to 0} \Delta s/\Delta t = ds/dt$
   where v = instantaneous speed
3. $s = \int v dt$ from t_1 to t_2
   where s = distance travelled during time t_1 to t_2
4. $v_{av} = (r_2 - r_1)/(t_2 - t_1)$Where v_{av} = average velocity
   r_2, r_1 = position vectors of a particle
   t_2, t_1 = time instants

5. $v = \lim_{\Delta t \to 0} \Delta r/\Delta t = dr/dt$
6. $v = |dr|/dt = ds/dt$Where v = magnitude of velocity
   s = displacement, for small time intervals magnitude of displacement will be equal to distance.
7. $a_{av} = (v_2 - v_1)/(t_2 - t_1)$
8. $a = \lim_{\Delta t \to 0} \Delta v/\Delta t = dv/dt$
   For Motion in straight line
9. Velocity is $v = dx/dt$
10. $a = dv/dt$
11. acceleration is $a = dv/dt = d^2x/dt^2$ For constant acceleration
12. $v = u+at$
13. x = distance moved in time t = $ut+\frac{1}{2}at^2$
14. $v^2 = u^2+2ax$ For motion in plane
15. Time of flight of the projectile = $(2u \sin \theta)/g$
16. $OB = (u^2 \sin 2\theta)/g$
17. $t = (u \sin \theta)/g$ At t vertical component of velocity is zero.
18. Maximum height reached by the projectile (in $t = (u \sin \theta)/g$) = $(u^2 \sin^2\theta)/2g$
19. V(B,S) = V(B,S')+V(S',S)
    Where
    - V(B,S) = velocity of body wrt to S)
    - V(B,S') = velocity of body wrt to S')
    - V(S',S) = velocity of S' wrt to S)
      we can rewrite above equation as
20. V(B,S') = V(B,S)- V(S',S)

Analytical treatment of projectile motion

Consider a particle projected with a velocity u of an angle θ with the horizontal earth's surface. If the earth did not attract a particle to itself, the particle would describe a straight line, on account of attraction of earth, however, the particle describes a curve path Let us take origin at the point of projection and x-axis along the surface of earth and perpendicular to it respectively shown in figure Here gravitational force is the force acting on the object downwards with constant acceleration of g downwards. There if no force along horizontal direction hence acceleration along horizontal direction is zero



Motion in x –direction

Motion in x-direction with uniform velocity
At t = 0, $X_0 = 0$ and u_x
$= u\cos\theta$

Position after time t , $x = x_0 + u_x t$
$X = (u\cos\theta)\, t$ -----eq(1)
Velocity at t , $V_x = u_x$
$V_x = u\cos\theta$ ------eq(2)

Motion in y-direction:

Motion in y-direction is motion with uniform acceleration
When $t = 0$, $Y_0 = 0$, $u_y = u\sin\theta$ and $a_y = -g$
After time 't', $V_y = y_y + a_y t$
$V_y = u\sin\theta - gt$ -----eq(3)

$$Y = Y_0 + u_y t + 1/2\, a_y t^2 ---(4)$$