

Electromagnetic Waves

The wave equation for light propagating in x-direction in vacuum may be written as

$$E = E_0 \sin \omega(t-x/c)$$

Where E is the sinusoidally varying electric field at the position x at time t. c is the speed of light in vacuum. The electric field is in the Y-Z plane. It is perpendicular to the direction of propagation of the wave.

There is a sinusoidally varying magnetic field associated with the electric field when light is propagating. This magnetic field is perpendicular to the direction of wave propagation and the electric field E.

$$B = B_0 \sin \omega(t-x/c)$$

Such a combination of mutually perpendicular electric and magnetic fields constitute an electromagnetic wave in vacuum.

Maxwell developed the theory of electromagnetic wave.

Maxwell's displacement current

Maxwell generalised Ampere's law to

$$\oint B \cdot dl = \mu_0(i + i_d)$$

$$i_d = \epsilon_0 \left(\frac{d\Phi_E}{dt} \right)$$

Where

Φ_E = the flux of the electric field through the area bounded by the closed curve along which the circulation of B is calculated.

Maxwell termed i_d as displacement current

I which is current due to flow of charges is often called conduction current. Ampere's law without Maxwell's modification is not valid if the electric field at the surface varies with time. The book gives the example of the region between capacitor plates to prove this point.

Continuity of electric current

Consider closed surface enclosing a volume a conductor in the shape of a cylinder. If all the

charge entering it is leaving it, the conduction is continuous. But if some charge is accumulated inside the volume, this continuity breaks.

However, if we consider both conduction current as well as displacement current, the total current is still continuous.

Maxwell's Equations

Gauss's laws for electricity and magnetism, Faraday's law and Ampere's are collectively known as Maxwell's equations

Gauss's law of electricity

$$\oint \mathbf{E} \cdot d\mathbf{s} = q / \epsilon_0$$

Gauss's law for magnetism

$$\oint \mathbf{B} \cdot d\mathbf{s} = 0$$

Faraday's law

$$\oint \mathbf{E} \cdot d\mathbf{l} = -d\Phi_B/dt$$

Ampere's law

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(i + i_d)$$

$$i_d = \epsilon_0 \cdot (d\Phi_E/dt)$$

These equations are satisfied by a plane electromagnetic wave given by

$$E_y = E = E_0 \sin \omega(t-x/c)$$

$$B_z = B = B_0 \sin \omega(t-x/c)$$

In proving these equations we come to the formula that

$$c = 1/\sqrt{\mu_0 \epsilon_0}$$

the value calculated from this expression comes out to be 2.99793×10^8 m/s which was same as the experimentally measured value of speed of light in vacuum. This also provides a confirmatory proof that light is an electromagnetic wave.

Energy Density and Intensity

For the waves described by

$$E_y = E = E_0 \sin \omega(t-x/c)$$

$$B_z = B = B_0 \sin \omega(t-x/c)$$

In any small volume dV , the energy of the electric field is

$$U_E = \frac{1}{2} \epsilon_0 E^2 dV$$

And the energy of the magnetic field is

$$U_B = B^2 dV / 2\mu_0$$

Total energy is

$$U = \frac{1}{2} \epsilon_0 E^2 dV + B^2 dV / 2\mu_0$$

When we substitute the values of E and B in the above equation and take an average over a longer period of time

$$u_{av} = \frac{1}{2} \epsilon_0 E_0^2 = B_0^2 / 2\mu_0$$

Intensity

The energy crossing per unit area per unit time perpendicular to the direction of propagation is called the intensity of a wave.

Consider a cylindrical volume with area of cross section A and length $c\Delta t$ along the X -axis (the direction of propagation of the light wave).

The energy contained in this cylinder $U = u_{av}(c\Delta t)A$

As intensity is per unit area per unit time $I = U/A\Delta t = u_{av}c$.

In terms of maximum electric field (substituting the value of $u_{av}c$)

$$I = \frac{1}{2} \epsilon_0 E_0^2 c$$

Momentum

The electromagnetic wave also carries linear momentum with it. The linear momentum carried by the portion of wave having energy U is given by $p = U/c$

If the wave incident on a material surface is completely absorbed, it delivers energy U and momentum $p = U/c$ to the surface. Hence electromagnetic waves incident on a surface exert a force on the surface.

Electromagnetic Spectrum

Maxwell's equations are applicable for electromagnetic waves of all wavelengths. Basic source of electromagnetic waves is an accelerated charge. Radio waves (used in radio and TV communication) are produced by charges accelerating in AC circuits having an inductor and capacitor.

Microwaves (used for radar communication and in cooking) are also produced by such electric circuits with oscillating current. Infrared waves (used in physical therapy) are produced by the atoms and molecules of hot bodies.

Visible light is also emitted by atoms under suitable conditions.

Ultraviolet radiation is emitted by atoms due to acceleration of electrons in the atom (the mechanism is same for visible light emission also).

The sun emits large amount of UV radiation. UV radiation is harmful to humans if absorbed in large amount.

X-rays (used in medical diagnosis) are produced when fast moving electrons collide with a metal target and decelerate. They are also harmful to living tissues.

Gamma rays are radioactive emissions from nuclei and have the shortest wavelengths among electromagnetic waves.

Radiation in atmosphere

Atmosphere up to 12 Km is called troposphere. Most of the water droplets, vapour and ice particles forming clouds are contained in this layer. Density of air at the top of the layer is about one tenth of the density near the surface.

Atmosphere between the 12 km and 50 km above earth's surface is called stratosphere. In the upper part of stratosphere there is ozone layer. Density of air at the top of this layer is about 10^{-3} times the density near the surface.

Atmosphere between the 50 and 80 km above earth's surface is called mesosphere. Above the mesosphere is ionosphere.

Sun sends electromagnetic waves of different wavelengths towards the earth.

Visible light is weakly absorbed by the atmosphere. Most of the infrared radiation is absorbed by the atmosphere and it gets heated. Ozone layer absorbs the ultraviolet radiation from sun and converts it into infrared radiation which is absorbed by the atmosphere. It is suspected that

ozone layer is slowly getting depleted and this is causing a great concern to scientists and especially environmental scientists and people who are promoting actions to protect environment – environmentalists.

Formula

Electromagnetic waves formula revision

1. The wave equation for light propagating in x-direction in vacuum may be written as

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2. Maxwell generalised Ampere's law to

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Ampere's law

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0(i + id)$$

$$id = \epsilon_0 \cdot (d\Phi_E/dt)$$

These equations are satisfied by a plane electromagnetic wave given by

$$E_y = E = E_0 \sin \omega(t-x/c)$$

$$B_z = B = B_0 \sin \omega(t-x/c)$$

$$4. \quad c = 1/\sqrt{\mu_0 \epsilon_0}$$

the value calculated from this expression comes out to be 2.99793×10^8 m/s which was same as the experimentally measured value of speed of light in vacuum. This also provides a confirmatory proof that light is an electromagnetic wave.

5. Total energy of the electromagnetic wave is

$$U = \frac{1}{2} \epsilon_0 E^2 dV + \frac{B^2 dV}{2\mu_0}$$

When we substitute the values of E and B in the above equation and take an average over a longer period of time

$$u_{av} = \frac{1}{2} \epsilon_0 E_0^2 = B_0^2 / 2\mu_0$$

6. intensity of electromagnetic wave is per unit area per unit time $I = U/\Delta t = u_{av}c$.

In terms of maximum electric field (substituting the value of $u_{av}c$)

$$I = \frac{1}{2} \epsilon_0 E_0^2 c$$

7. The electromagnetic wave also carries linear momentum with it. The linear momentum carried by the portion of wave having energy U is given by

$$p = U/c$$

where U = energy contained in the portion of the wave.