

Charge and Electric Field



Charge and Electric Field

Charge:

Due to gravitational force an electron is expected to attract another electron with a force of 5.5×10^{-67} N.

But, an electron is found to repel another electron at 1 cm with a force of 2.3×10^{-24} N. This extra force is called the electric force.

The electric force is very large compared to the gravitational force. The electrons must have some additional property apart from their mass, which is responsible for this electric force. This property is termed as charge.

Electric Force

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Two kinds of charges

As a convention, the charge on proton is termed positive and the charge on electron is termed negative. The charges on proton and electron have the same strength but are of different nature. Two protons repel each other, two electrons repel each other, but a proton and an electron attract each other.

Units of charge: The SI unit of charge is coulomb abbreviated as C. 1 coulomb is defined as the charge flowing through a wire in 1 second if the electric current in it is 1 A.

The charge on a proton is

$$e = 1.60218 \times 10^{-19} \text{ C}$$

The charge on an electron is the negative of this value.

Charge is quantized:

If protons and electrons are the only charge carriers in the universe, the charge on any object has to be in multiples of e only. So charge is quantized.

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Charge is conserved:

It is not possible to create or destroy net charge, even though we can destroy or create charged particles. For example in a beta decay process, a neutron converts itself into a proton and a fresh electron is created. Thus the net charge however remains zero before and after the event.

Frictional electricity:

When a glass rod is rubbed with a silk cloth, electrons are transferred from the glass rod to the silk cloth. Due to excess electrons, silk cloth becomes negatively charged and due to excess protons, glass rod becomes positively charged. Thus due to friction, both bodies get charged.

Induction:

A redistribution of charge in a material due to the presence of a nearby charged body, is called induction.

Electric force and the Coulomb's law:

Coulomb established the mathematical formula for the electric force between two charges.

$$F = k \cdot q_1 \cdot q_2 / r^2$$

q_1, q_2 charges

r = separation between charges

k = constant In SI units k is measured to be $8.98755 \cdot 10^9 \text{ N-m}^2/\text{C}^2$

The constant k is often written as $1/4\pi\epsilon_0$.

The constant ϵ_0 is called the permittivity of the space and its value is

$$\epsilon_0 = 8.85419 \cdot 10^{-12} \text{ C}^2/\text{N-m}^2$$

Electric Field:

A charge produces an electric field in the space around it and this electric field exerts a force on any charge placed on it.

The intensity of field is defined as $\text{Vector } E = \text{Vector } F/q$

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We can interpret that intensity of field is force exerted by a charge (q_1) on another charge (q_2) per unit charge of the charge on which field is exerting the force (q_2). Hence force exerted on a charge in an electric field is equal to intensity of electric field multiplied by the charge on which the force is exerted.

Electric field is a vector quantity. It has magnitude and direction.

Lines of electrical force:

The electric field in a region can be graphically represented by drawing lines. These lines are called lines of electric force or electric field lines. Lines of force are drawn in such a way that the tangent to a line of force gives the direction of the resultant electric field there.

For a point charge, electric field lines are straight lines originating from the charge in all directions.

Electric potential energy:

Electric potential energy comes into picture as the configuration of the system of charges changes. As, in the system, charges exert electric forces on each other, if the position of one or more charges is changed, work needs to be done by the system or by the environment.

If work is done by the system, the change in potential energy is $-W$. This is so because if system does the work its potential energy decreases.

If we take the potential energy of the system to be zero when one of the charges is at an infinite distance or separation, the potential energy when the charge is brought up to a separation of r will be

$$U(r) = U(r) - U(\infty) = q_1 q_2 / 4\pi\epsilon_0 r.$$

The potential energy will have units of work or force*distance. That is why in the denominator only r is there. In force formula r^2 term is there.

If two positive charges are close together there is repulsion among them. Hence there is a potential energy in them. When they are very far apart the potential energy between the two positive charges is zero. Hence the assumption that potential energy of a system is zero when the charges are at an infinite distance or separation is an appropriate assumption.

Electric potential:

Electric field (which creates electric force) can also be described by assigning a scalar quantity V at each point. This scalar quantity is termed as electric potential.

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If a test charge is moved in an electric field from a point A to a point B while all the other charges in question remain fixed, if the electric potential energy of the system changes by $U_b - U_a$, we define the potential difference between the point A and point B as

$$V_b - V_a = (U_b - U_a)/q$$

Potential difference is equal to change in potential energy for unit charge between two points A and B.

You can calculate potential energy difference between points A and B by multiplying potential difference by the charged moved from A to B.

Relation between electric field and potential

Scalar product of Field vector and displacement vectors gives potential.

Electric field (E) is force being applied by the charge divided by the test charge.

Potential is work done per unit charge.

So we can interpret it as Vector Force * vector distance/ $q = qE \cdot r/q = E \cdot r$ which is scalar product of field vector and displacement vectors.

Hence we can calculate potential V if we know E and r.

If we know V we can find E through the relation

$$E_x = -\partial V/\partial x$$

$$E_y = -\partial V/\partial y$$

$$E_z = -\partial V/\partial z$$

We can find the x,y and z components of E.

E can be written as $E_x \mathbf{i} + E_y \mathbf{j} + E_z \mathbf{k}$

dV can also be written as $-E dr \cos \theta$ where θ is the angle between the field E and the small displacement dr .

If θ is equal to zero, $dV = -E dr$ or $-dV/dr$ is maximum. Thus the electric field is along the direction in which the potential decreases at the maximum rate.

The potential does not vary in a direction perpendicular to the electric field as $\cos \theta = 0$.

If we draw equipotential surface around a charge, component of the electric field parallel to an equipotential surface is zero as potential does not change along the surface. Electric field is perpendicular to the surface at the any point on the surface.

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For a point charge, the electric field is radial and the equipotential surfaces are concentric spheres with centres at the charge.

Electric Dipole:

A combination of two charges $+q$ and $-q$ separated by a small distance of d constitutes an electric dipole.

Electric dipole moment:

It is defined as a vector $p = q \times \text{distance vector } d$

where distance vector is the vector joining the negative charge to the positive charge.

The line along the direction of the dipole moment is called the axis of the dipole.

Electric potential due to a dipole at a point P

Point is at distance r from the centre of the dipole ($d/2$) and the line joining the point P to the centre of the dipole make an angle θ with the direction of dipole moment (from $-q$ to $+q$)

Potential at P due to charge $-q = - [1/4\pi\epsilon_0][q/(r + (d\cos \theta)/2)]$

Potential at P due to charge $q = [1/4\pi\epsilon_0][q/(r - (d\cos \theta)/2)]$

Net potential due to q and $-q = [1/4\pi\epsilon_0](qd \cos \theta)/r^2$

qd can be replaced by p .

The general definition of electric dipole

$$V = p \cos \theta / 4\pi\epsilon_0 r^2$$

where p is magnitude of electric dipole moment defined above

Any charge distribution that produces electric potential given by above formula is called an electric dipole. The two charge system can be expressed as $p = qd$.

The potential at a distance r from a point charge q is given by $V = [1/4\pi\epsilon_0](q/r)$

The potential at a distance r from an electric dipole with electric dipole moment of p is given by $V = [1/4\pi\epsilon_0](p \cos \theta)/r^2$

Electric field due to a dipole

$$E_r = [1/4\pi\epsilon_0](2p \cos \theta)/r^3$$

$$E_\theta = [1/4\pi\epsilon_0](p \sin \theta)/r^3$$

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Resultant electric field at P = $E = \sqrt{E_r^2 + E_\theta^2}$

$$= \frac{1}{4\pi\epsilon_0} \left(\frac{p}{r^3} \right) \sqrt{3 \cos^2 \theta + 1}$$

The angle the resultant field makes with radial direction OP (O is the centre point of the dipole axis and P is that at which electric field is being calculated) is α .

$$\tan \alpha = E_\theta / E_r = \frac{1}{2} \tan \theta \text{ or}$$

$$\alpha = \tan^{-1} \left(\frac{1}{2} \tan \theta \right)$$

Special cases

a. $\theta = 0$. In this case P is on the axis of the dipole. This position is called an end-on position.

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{p \cos \theta}{r^2} \right) \text{ as } \theta = 0$$

$$V = \frac{1}{4\pi\epsilon_0} \left(\frac{p}{r^2} \right)$$

$$\text{General formula for } E = \frac{1}{4\pi\epsilon_0} \left(\frac{p}{r^3} \right) \sqrt{3 \cos^2 \theta + 1} \text{ as } \theta = 0$$

$$E = \frac{1}{4\pi\epsilon_0} \left(\frac{2p}{r^3} \right)$$

b. $\theta = 90^\circ$. In this case P is on the perpendicular bisector of the dipole axis.

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$$E = \frac{1}{4\pi\epsilon_0} \left(\frac{p}{r^3} \right)$$

Angle α is given by $\tan \alpha = \tan 90^\circ / 2 = \infty$ Therefore $\alpha = 90^\circ$.

Torque on an electric dipole placed in an electric field.

If the dipole axis makes an angle θ with the electric field magnitude of the torque = $|\Gamma| = pE \sin \theta$

In vector notation $\Gamma = p \times E$ Potential energy of a dipole placed in a uniform electric field dipole axis makes an angle θ with the electric field magnitude of the torque

$$\text{Change in potential energy} = U(\theta) - U(90^\circ) = -pE \cos \theta - 0 = -pE \cos \theta$$

If we choose the potential energy of the dipole to be zero when $\theta = 90^\circ$, above equation becomes

$$U(\theta) = -pE \cos \theta = -p \cdot E$$

Electric field inside a conductor

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There can be no electric field inside a conductor in electrostatics.

When electric field is applied from left to right some free electrons move toward the left creating a negative charge on the left surface. Due to which there will be positive charge on the right surface. Due to this charge buildup, coulomb attraction sets between these two charges and an electric field opposite in direction to the applied electric field is set up. The movement of free electrons continues till the applied electric field and the electric field due to redistribution of electrons are equal. Hence inside the conductor these two electric fields balance each other and there is no electric field inside a conductor in electrostatics.

Conductors, insulators, semiconductors

Conductors have free electrons that move throughout the body. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. The free electrons are called conduction electrons in this context.

In insulators, electrons are tightly bound to their respective atoms or molecules. So in an electric field, they can't leave their parent atoms. They are insulators or dielectrics.

In semiconductors, at 0 K there are no free electrons but as temperature raises, small number of free electrons appear (they are able to free themselves from atoms and molecules) and they respond to the applied electric field.