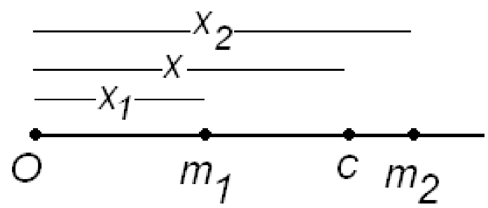
CENTRE OF MASS AND CIRCULAR MOTION

As shown in figure consider two particles having mass m_1 and m_2 lying on X-axis at



distance of x_1 and x_2 respectively from the origin (O). The centre of mass of this system is that point whose distance from origin O is given by

$$x = m_1x_1 + m_2x_2/\ m_1 + m_2$$

Here, x is the mass-weight average position of x_1 and x_2. The centre of mass of the two particles of equal mass lies at the centre (on the line joining the two particles between the two particles)

Consider a set of n particles whose masses are m_1 m_2, m_3, … mn and whose vector relative to an origin O are r_1, r_2, r_3, …….. rn respectively The centre of mass of this set of particles is defined as the point with position vector r_{CM}

$$\vec{r}_{CM} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots + m_n\vec{r}_n\ /m_1 + m_2 + m_3 + \cdots + mn$$

$$\vec{r}_{CM} = \sum{}^{n}{}_{i}{=}1\ m_i\vec{r}_i/\ M$$

Here M is the total mass of the body.

Centre of mass of continuous bodies

For calculating centre of mass of continuous body, we first divide the body into suitably chosen infinitesimal elements. The choice is usually determined by the symmetry of the body.

Consider an element dm of the body having position vector r, the quantity miri can be replaced by dmri , direct sum over particles becomes integral over the body

$$\vec{r}CM = 1/\ M \int \vec{r}dm$$

In component form, this equation can be written as

$$xcm = 1/\ M \int x\ dm$$

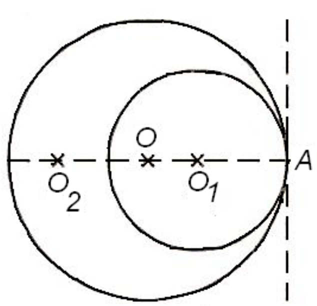
$$ycm = 1/\ M \int y\ dm$$

$$zcm = 1\ /M \int z\ dm$$

To evaluate the integral we must express the variable m in terms of spatial coordinates x, y, z or r

Solved Numerical

Q) A circular plate of uniform thickness has a diameter of 56 cm. A circular portion of diameter 42cm is removed from one edge of the plate as shown in figure. Find the centre of mass of the remaining portion



Let O be the centre of circular plate and O_1, the centre of circular portion removed from the plate. Let O_2 be the centre of mass of the remaining part. Area of original plate = $\pi R_2 = (28)_2 \pi$ cm_2 Area removed from circular plate = $\pi r_2 = (21)_2\ \pi\ cm_2$ Let σ be the mass per cm_2 . Then Mass of the original plate m = $(28)_2\pi\sigma$ Mass of the removed part $m_1 = (21)_2\ \pi\sigma$ Mass of the remaining part $m_2 = (28)_2\pi\sigma - (21)_2\ \pi\sigma = 343\pi\sigma$ Now the masses m_1 and m_2 may be supposed to be concentrated at O_1 and O_2 respectively. Their combined centre of mass is at O. Taking O as origin we have form definition of centre of centre of mass.

$$xcm = m_1x_1 + m_2x_2\ /m_1 + m_2$$

$x_1 = OO_1 = OA - O_1A = 28 - 21 = 7cm$
$x_2 = OO_2 = ?$, xcm = 0

$$0 = (21)^2\pi\sigma \times 7 + 343\pi\sigma \times x_2 / m_1 + m_2$$

$$(21)_2\pi\sigma \times 7 + 343\pi\sigma \times x_2 = 0 \quad x_2 = -9 \text{ cm}$$

Difference between Centre of mass (CM) and centre of gravity (CG)
The center of gravity is based on weight, whereas the center of mass is based on mass. So, when the gravitational field across an object is uniform, the two are identical. However, when the object enters a spatially-varying gravitational field, the CG will move closer to regions of the object in a stronger field, whereas the CM is unmoved. More practically, the CG is the point over which the object can be perfectly balanced; the net torque due to gravity about that point is zero. In contrast, the CM is the average location of the mass distribution. If the object were given some angular momentum, it would spin about the CM. Clearly if gravitational acceleration is uniform rCm = rCG If gravitational field is not uniform rCm ≠ rCG

Velocity of Centre of mass

$$\vec{v}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v} \;/m_1 + m_2 + m_3 + \cdots + mn$$

Momentum of centre of mass

$$\vec{P} = M\vec{v}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n$$

$$\vec{P} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \cdots + \vec{P}_n$$

Acceleration of centre of mass

$$\vec{a}_{CM} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a} \;/ m_1 + m_2 + m_3 + \cdots + mn$$

Force on centre of mass

$$\vec{F} = M\vec{a}_{CM} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a}_n$$

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}n$$

Equation shows that the system moves under the influence of the resultant external force F as if the whole mass of the system is concentrated at its centre of mass

Law of conservation of momentum

$$\vec{P} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \cdots + \vec{P}_n$$

Above equation shows that “If resultant external force acting on a system of particle is zero, then the total linear momentum of the system remain constant” this statement is known as the law of conservation of linear momentum.

Solved numerical

Q) A man weighing 70kg is standing at the centre of a flat boat of mass 350 kg. The man who is at a distance of 10m from the shore walks 2m towards it and stops. How far will he be from the bank? Assume the boat to be of uniform thickness and neglect friction between boat and water.

Solution

Man and boat form a system. This system is not acted by any external force Thus according to law of conservation of momentum Centre of mass of system will remain unchanged with reference to observer on the bank Now man is standing at the centre of boat thus CM is at 10 m from bank

$x_{CM} = m_1 x_1 + m_2 x_2 \,/\, m_1 + m_2$

$10 = 70x_1 + 350x_2 \,/\, 70 + 350$

420= 7x1 + 35 x2
60= x1 + 5x2 ---- eq(1)
Since man has walked 2 m distance between the CM of Boat and Man is 2m
Also $x_1 – x_2 = {}_2$ -----eq(2)
From equation (1) and (2) we get
X_1 = 25/3=8.33 m