Allegations and Mixtures

THEORY

In the chapter on Averages, we had seen the use of the weighted average formula. To recollect, the weighted average is used when a number of smaller groups are mixed together to from one lager group.

If the average of the measured quantity was

A_1 for group	1	containing	n_1	elements
A_2 for group	2	containing	n_1	elements
A_3 for group	3	containing	n_3	elements
A_k for group	k	containing	n_k	elements

We say that weighted average, Aw is given by:
$Aw = (n_1A_1 + n_2A_2 + n_3A_3 + \ldots\ldots + n_kA_k)/(n_1 + n_2 + n_3 \ldots. + n_k)$

That is, the weighted average

= Sum total of all groups/total number of elements in all groups together

In the case of the situation where just two groups are being mixed, we can write this as:

$$Aw = ((n_1A_1 + n_2A_2)\ (n_1 + n_2)$$

Rewriting this equation we get: $((n_1 + n_2)\ Aw = n_1A_1 + n_2A_2$

$$n_1\ (Aw - A_1) = n_2\ (A_2 - Aw)$$

or $n_1/\ n_2 = (A_2 - Aw)/(Aw - A_1)$ → the allegation equation.

The allegation Situation

Two group of elements are mixed together to from a third group containing the elements of both the groups.

If the average of the first group is A_1 and number of elements is n_1 and average of the second group is A_2 and the number of elements is n_2 then to find the average of the new group formed, we can use either the weighted average equation or the allegation equation.

As a convention, we take $A_1 < A_2$. Then, by the principal of average, we get $A_1 < Aw < A_2$.

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The Straight Line Approach

As we have seen, the cross method becomes quiet cumbersome in Case 2 and Case 3. We will now proceed to modify the cross method so that the question can be solved graphically in all the three cases.

SOME TYPICAL SITUATIONS WHERE ALLIGATIONS CAN BE USED

Given below are typical alligation situations, which students should be able to recognize. This will help them improve upon the time required in solving questions. Although in this chapter we have illustrated problems based on allegation at level 1 only, allegation is used in more complex problems where the weighted averages is an intermediate step in the solution process.

Some Keys to stop A_1, A_2 and Aw and differentiate these from n_1 and n_2

1. Normally, there are 3 average mentioned in the problem, while there are only 2 quantities. This isn't foolproof thought, since at times the question might confuse the student by given 3 values for quantities representing $n_1 + n_2$ respectively.
2. A_1, A_2 and Aw are always rate units, while n_1 and n_2 are quantities units.
3. The denominator of the average unit corresponds to the quantity units.
4. All percentage values represent the average values.

A typical problem

A typical problem related to the topic of allegation goes as follows:

> 4 liters of wine are drawn from a cask containing 40 liters of wine. It is replaced by water. The process is repeated 3 times
>
> (a) What is the final quantity of wine left in the cask.
> (b) What is the ratio of wine to water finally.

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If we try to chart out the process, we get: out of 40 liters of wine, 4 are drawn out.

This leaves 36 liters wine and 4 liters water.(ratio of 9 : 1)