

Rotational Mechanics



Rotation of a rigid body

If each particle of a rigid body moves in a circle, with centres of all the circles on a straight line and with planes of the circles perpendicular to this line we say that the body is rotating about this line. The straight line is called the axis of rotation. (Particle makes circular motion. Rigid body makes rotation.)

Kinematics of rotation of rigid body

For a rigid body let the axis of rotation be Z-axis.

At time $t = 0$, let particle P be at P_0 .

Perpendicular to axis of rotation from P_0 be PQ. (Q is on the axis)

If at time t Particle P moves P_1 and angle $P_0Q P_1 = \theta$.

Hence the particle has rotated through θ .

All particles have rotated through θ .

We can say the whole rigid body has rotated through angle θ .

The angular position of the body at time t is θ .

If P has made a complete revolution its circular path, every particle will do so and hence rigid body has done so. We can say rigid body made a complete revolution and it has rotated through an angle of 2π radians.

Hence, rotation of a rigid body is measured by the rotation of a line PQ (P is a particle on the rigid body and Q is a point on the axis of rotation and PQ is perpendicular from P to the axis of rotation).

As the rotation of rigid body is defined in terms of the circular motion of a particle on the rigid body, kinematics of circular motion of particle becomes applicable to rotation.

Angular variables

Rotational Mechanics



θ = angular position of the particle

ω = angular velocity = $d\theta/dt = \lim_{\Delta t \rightarrow 0} \Delta\theta/\Delta t$

α = angular acceleration = $d\omega/dt = d^2\theta/dt^2$

If the angular acceleration is constant, formulas similar in form to linear formulas can be used to find the angular variables:

$$\theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2 \alpha \theta$$
Where

ω_0 is velocity at time $t = 0$.

Given the axis of rotation, the body can rotate in two directions – clockwise or anticlockwise. One of the directions has to be defined as positive direction according to the convenience of the problem.

The SI unit for angular velocity is radian/sec (rad/s).

$$\text{One revolution/sec} = 2\pi \text{ radian/sec}$$

Similar to the circular motion of the particle, in rotation for a particle P

s = Linear distance traveled by the particle in circular motion

Δs = Linear distance traveled by the particle in circular motion in time Δt

$$\Delta s = r\Delta\theta$$

Where

r = radius of the circle over which the particle is moving

$\Delta\theta$ = angular displacement in time Δt

$$\Delta s/\Delta t = r\Delta\theta/\Delta t$$

$v = r\omega$ where v = linear speed of the particle

Rotational Mechanics



a_t = rate of change of speed of the particle in circular motion

$$a_t = dv/dt = r d\omega/dt = r \alpha$$

Rotational dynamics

In rotation of a body the resultant force due to external forces is zero, but the resultant of

Torque produced by the external forces is nonzero and this torque produces rotation motion.

Torque of a force about the axis of rotation

First we define torque a force about a point.

For a force F acting on a particle P , to find torque about a point O , define the position vector of P with respect to O . Let this position vector be r .

$$\text{Torque of force } F \text{ about } O = \Gamma = F \times r.$$

This is vector product of two vectors hence a vector quantity, as per the rules of vector product, the direction of Γ will be perpendicular to F and r .

When the torque about an axis of rotation is to be determined, select a point on the axis of the rotation and find the torque of the force acting on a particle about this point. Find the angle between the axis and the line joining the point on axis (about which the torque is calculated) and the particle (one which the force is acting). Let the angle be θ .

Torque about the axis due to a force is the component along the axis, of the torque of the force about a point on the axis.

$$\text{Magnitude of the torque} = |F \times r| \cos \theta$$

Rotational Mechanics



The torque about the axis is same, even if different points are chosen along the axis for determining the torque of the force about those points.

Some special cases of relation between force and the axis of rotation.

1. Force is parallel to the axis of rotation.

Torque along the axis is zero.

2. F and r are collinear. The torque about O is zero and the torque about axis is zero.

3. Force and axis are perpendicular but they do not intersect. In three dimensions, two lines may be perpendicular without intersecting. Example: A vertical line on a wall and a horizontal line on the opposite wall.

In this case torque about the axis is equal to Force multiplied by the perpendicular to axis from the force direction (line along which the force is acting).

Torque produced by forces on a particle

A particle having circular motion will have two forces acting on it.

One force produces tangential acceleration dv/dt in it. Hence the force named tangential force is ' ma ' = $mdv/dt = mr\alpha$

This force creates a torque of $mr^2\alpha$

the other force creates radial acceleration or centripetal acceleration $\omega^2 r$. Hence the force named radial force is $m\omega^2 r$

As intersects the axis of rotation, the torque produced by it is zero.