

Gauss's Law and Potential

Gauss's law and

Gauss's law's application in simple cases, such as, to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell

Flux of an electric field

If in a plane surface area of Δs , a uniform electric field E exists, and makes an angle θ with the normal to the surface area (positive normal - you can arbitrarily decide which direction is positive), the quantity

$$\Delta\Phi = E \Delta s \cos \theta$$

is called the flux of the electric field through the chosen surface.

Δs is represented as a vector. Also it is area.

$$\Delta\Phi = E \cdot \Delta s$$

Where E and Δs are vectors and $\Delta\Phi$ is a scalar quantity.

As flux is a scalar quantity, it can be added arithmetically. Hence surfaces which are not on single plane, can be divided into small parts which are plane, the flux through each part can be found out and the sum is flux through the complete surface.

Non-uniform electric field can also be tackled that way. Divide the surface into parts over which the electric field is uniform and then find the flux in each part and sum them.

Using the techniques of integration flux over a surface is:

$\Phi = \int E \cdot \Delta s$ When integration over a closed surface is done a small circle is placed on the integral sign. ($\oint$).

Flux over a closed surface

$$\Phi = \oint E \cdot \Delta s \text{ (* Intregation over a closed surface)}$$

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Solid angle:

Typical example is the angle in the paper containers used by moongfaliwalas.

$$\Omega = S/r^2$$

S = the area of the part of sphere intercepted by the cone

r = radius of the sphere assumed on which we are assuming the cone

A complete circle subtends an angle 2π

Any closed surface subtends a solid angle 4π at the centre.

How much is the angle subtended by a closed plane curve at an external point?
Zero.

Gauss's law

The flux of the net electric field through a closed surface equals the net charge enclosed by the surface divided by ϵ_0 .

In symbols

$\oint E \cdot \Delta s$ (* Integregation over a closed surface) $= q_{in}/\epsilon_0$. Where q_{in} = charge enclosed by the closed surface ϵ_0 = emittivity of the free space

It needs to be stressed that flux is the resultant of all charges existing in the space. But, its quantity is given the right hand side.

Applications of Gauss's law

a. Charged conductor

The free electrons redistribute themselves to make the field zero at all the points inside the conductor. Any charge injected anywhere in the conductor must come over to the surface of the conductor so that the interior is always charge free.

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If there is cavity inside the conductor, for example a hollow cylinder and a charge $+q$ is placed in this cavity, as the inside of the conductor has to be charge free, negative charge appears on the inside of the cavity. If the conductor is neutral, a charge $+q$ will appear on the surface.

b. Electric field due to a uniformly charged sphere

A total charge Q is uniformly distributed in a spherical volume of radius R . what is the electric field at a distance r from the centre of the charge

distribution outside the sphere?

Through Gauss's law we get $E 4 \pi r^2 = Q / \epsilon_0$.

$$\text{So } E = Q / 4 \pi \epsilon_0 r^2$$

The electric field due to a uniformly charged sphere at a point outside it, is identical with the field due to an equal point charge placed at the centre.

Field at an internal point

At centre $E = 0$

At any other point r less than R radius of the sphere

$$E = Qr / 4 \pi \epsilon_0 R^3$$

c. Electric field due to a linear charge distribution

The linear charge density (charge per unit length) is λ .

Electric field at a distance r from the linear charge distribution

$$E = \lambda / 2 \pi \epsilon_0 r$$

d. Electric field due to a plane sheet of charge

Plane sheet with charge density (charge per unit area) σ .

Field at distance d from the sheet = $E = \sigma / 2 \epsilon_0$

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We see that the field is uniform and does not depend on the distance from the charge sheet. This is true as long as the sheet is large as compared to its distance from P.

e. Electric field due to a charged conducting surface

To find the field at a point near this surface but outside the surface having charge density σ .

$$E = \sigma / \epsilon_0$$

Notice in case of plane sheet of charge it $\sigma/2\epsilon_0$. But in the case of conductor is σ/ ϵ_0

Spherical Charge Distributions

Useful results for a spherical charge distribution of radius R

a) The electric field due to a uniformly charged, thin spherical shell at an external point is the same as that due to an equal point charge placed at the centre of the shell.

b) The electric field due to a uniformly charged, thin spherical shell at an internal point is zero.

c) The electric field due to a uniformly charged sphere at an external point is the same as that due to an equal point charge placed at the centre of the sphere.

d) The electric field due to a uniformly charged sphere at an at an internal point is proportional to the distance of the point from the centre of the sphere. Thus it is zero at the centre and increases linearly as one moves out towards the surface.

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e) The electric potential due to a uniformly charged, thin spherical shell at an external point is the same as that due to an equal point charge placed at the centre of the shell. $V = Q/4\pi\epsilon_0 r$

f) The electric potential due to a uniformly charged, thin spherical shell at an internal point is the same everywhere and is equal to the that at the surface. $V = Q/4\pi\epsilon_0 R$

g) The electric potential due to a uniformly charged sphere at an external point is the same as that due to an equal point charge placed at the centre of the sphere. $V = Q/4\pi\epsilon_0 r$

Electric potential energy of a uniformly charged sphere

Charge density $\rho = 3Q/4\pi R^3$ (charge density is per unit volume)

Electric potential energy of the charged sphere = $3Q^2/20\pi\epsilon_0 R$

Electric potential energy of a uniformly charged, thin spherical shell

Electric potential energy of the thin spherical shell = $Q^2/8\pi\epsilon_0 R$

Earthing a conductor

The earth is good conductor of electricity.

Earth's surface has a negative charge of $1nC/m^2$. All conductors which are not given any external charge are also very nearly at the same potential.

But the potential of earth is often taken as zero.

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When a conductor is connected to earth, the conductor is said to be earthed or grounded and its potential will become zero.

In appliances, the earth wire is connected to the metallic bodies. If by any fault, the live wire touches the metallic body, charge flows to the earth and the potential of the metallic body remains zero. If it not connected to the earth, the user may get an electric shock.