Progressions

ARITHMETIC PROGRESSIONS

Quantities are said to be in arithmetic progression when they increase or decrease by a common difference.

This each of the following series forms an arithmetic progressions

3, 7, 11, 15.....

8, 2, -4 10,......

a, a + d , a + 2d, a + 3d,....

To Find the Sum of the Given Number of Terms in a arithmetic progressions

Let a denote the first term. *d,* the common difference, and *n* the total number of terms. Also, let *L* denote the last term, and S the required sum; then

$$S = n(a + L)/2$$

$$L = a + (n - 1) d$$

$$S = n/2 \times [2a + (n - 1)d]$$

To Find the Arithmetic Mean between any Two Given Quantities

Let *a* and *b* be two quantities and *A* be their arithmetic mean.
Then since *a, A, b* are in *AP.* We must have

$$b - A = A - a$$

Each being equal to the common difference;

This gives us $A = (a + b)/2$

To Insert a Given Number of Arithmetic Means between Two Given Quantities

Let a and b be the given quantities and n be the number of means.

Including the extremes the number of terms will then be n + 2 so that we have to find a series of n + 2 terms in *AP*, of which *a* is the first, and *b* is the last term.

Let *d* be the common difference;

Then be = the (n + 2)th term

= a +(n + 1)*d*

Hence, *d* = (b - a)/(n + 1)

And the required means are

A+ (b - a)/n -1, a + 2(b - a)/n + 1,… a + n(*b* - *a*)/ n + 1

GEOMETRIC PEOGRESSIONS

Quantities are said to be in Geometric progressions when they increase or decrease by a constant factor.

The constant factor is also called the *common ratio* and it is found by dividing any term by the term immediately preceding it.

If we examine the series $a, ar, ar^2, ar^3, ar^4, \ldots$

we notice that in any term the index of r is always less by one than the number of the term in the series.
If *n* be the number of terms and if l denote the last, or nth term, we have

$$l = ar^{n-1}$$

The Find the Geometric Mean between Two Given Quantities

Let a and b be the two quantities; G the geometric mean. Then since a, G, b are in GP,

b/G = Gla

Each being equal to the common ratio

$G^2 = ab$

Hence G = √ab

To Insert a given Number of Geometric Means between Two Given Quantities

Progressions

Let a and b the given quantities and n the required number of mean to be inserted. In all there will be n + 2 terms so that we have to find a series of n + 2 terms in GP of which a is first and b the last.

Let r be common ratio;

Then b = the(n + 2)th term = $ar^{n+1};$

$r^{(n+1)} = b/a$

` $r = (b/a)^{1/n+1}$

Hence the required number of mean are $ar, ar^2, \ldots. ar^n$,

where r has the value found in (1).

To Find Sum of a Number of Terms in a Geometric Progression

Let a be the first term, r the common ratio, n the number of terms, and S_n be the sum to n terms.

If $r > 1$, then

$$S_n = a(r^n - 1)/(r - 1)$$

If $r < 1$, then $S_n = a(1 - r^n)/(1 - r)$

To Find the Harmonic Mean between Two Give Quantities

Let a, b be the two quantities, H their harmonic mean; then 1/a, 1/H and 1/b are in A.P.;

$$1/H - 1/a = 1/b - 1/H$$

$$2/H = 1/a + 1/b$$

i.e. $H = 2ab/(a + b)$

THEOREMS RELATED WITH PROGRESSION

If A, G, H are the arithmetic, geometric, and harmonic means between a and b, we have

$$A = (a + b/2)$$

$$G = \sqrt{ab}$$

$$H = 2ab/(a + b)$$

Therefore, $A \times H = (a + b)/2 \times 2ab/(a + b) = ab = G^2$

that is, G is the geometric mean between A and H.

From these result we see that

$$A - G = a + b/2 - \sqrt{ab} = (a + b - 2\sqrt{ab})/2$$

$$= [(\sqrt{a} - \sqrt{b})/\sqrt{2}]^2$$