Inequalities

PROPERTIES OF INEQUALITIES

For any two real numbers a and b, only one of the following restrictions can hold true:

$$a = b, a > b \text{ or } a < b$$

PROPERTIES OF INEQUALITIES

1. If a > b then b < *a* and vice versa.
2. If a > b and b > then a > c.
3. If a > b then foe any c. a + c > b + c. In other words, an inequality remains true if the same number is added on both sides of the inequality.
4. Any number can be transposed from one side of an inequality to the other side of the inequality with the sign of the number reversed. This does not change the sense of the inequality.
5. If a > b and c > 0 then ac > bc. Both sides of an inequality may be multiplied (or divided) by the same positive number without changing the sense of the inequality.
6. If a > b and c > d then a + c > b + d. (Two inequalities having the same sense may be added termwise.)

Certain Important Inequalities

1. $a^2 + b^2 \geq 2ab$ (Equality for a = b)
2. $|a + b| \leq |a| + |b|$ (Equality reached if both a and b are of the same sign or if one of them is zero.) This can be generalized as $|a_1 + a_2 + a_3 + \ldots.+a_n| \leq |a_1|+|a_2|+|a_3|+\ldots..|a_n|$
3. $|a - b| \geq |a| - |b|$
4. $ax^2 + bx + c \geq 0$ if a > 0 and $D = b^2 - 4ac \leq 0$. The equality is achieved only if D = 0 and x = -b/2a.
5. Arithmetic mean ≥ Geometric mean. That is, $x = \frac{(a+b)}{2} \geq ab$

Inequalities

Some Important Result

- If $a > b$, then it is evident that

$$a + c > b + c$$

$$a - c > b - c$$

$$ac > bc$$

$$a/c > b/c; \text{ that is/}$$

an inequality will still hold after each side has been increased, diminished =, multiplied, or divided by the same positive quantity.

If $a - c > b$

By adding c to each side,

$a > b + c$; which shows that

in an inequality any term may be transposed from one side to the other if its sign is changed.

- If $a > b$, then evidently $b < a$; that is,
  if the sides of an inequality be transposed, the sign of inequality must be reversed.
- If $a > b$, then $a - b$ is positive, and $b - a$ is negative; that is , -a (-b) is negative, and therefore $-a < -b$; hence,

Definition of Solution of an Inequality

The solution of an inequality is the value of an unknown for which this inequality reduces to a true numerical identity. That is, to solve an inequality means to find all the values of the variable for which the given inequality is true.
An inequality has no solution if there is no such value for which the given inequality is true.

Equivalent Inequalities: two inequalities are said to be equivalent if any solution of one is also solution of the other and vice versa.
If both inequalities have no solution, then they are also regarded to be equivalent. To solve an inequality we use the basic properties of an inequality which have been illustrated above.