

Faraday's law of electromagnetic induction

Whenever the flux of magnetic field through the area bounded by a closed conducting loop changes, an emf is produced in the loop.

The emf is given by

$$\varepsilon = -d\phi/dt \dots (38.1)$$

Where $\phi = \int \mathbf{Vr}(\mathbf{B}) \cdot \mathbf{Vr}(d\mathbf{S})$ is the flux of the magnetic field through the area.

The SI unit of magnetic flux is called weber which is equivalent to tesla-metre.

Lenz's law

The direction of the induced current is such that it opposes the change that has induced it.

Origin of EMF

An electric current is established in a conducting wire when an electric field exists in it.

What is an electric field: A charge produces something called an electric field in the space around it and this electric field exerts a force on any charge (except the source charge itself) placed in it.

The flow of charge or movement of charge in response to an electric tends to destroy the field and some external mechanism is needed to maintain the electric field in the wire.

It is the work done per unit charge by this external mechanism that we call emf.

What is the mechanism that produces emf in induced emf?

The flux $\int \mathbf{B} \cdot d\mathbf{S}$ can be changed by

a. keeping the magnetic field constant as time passes and moving whole or part of loop.

- b. Keeping the loop at rest and changing the magnetic field
- c. Combination of both

Motional EMF

Example of a rod of length l moving through a constant magnetic field.

The free electrons in the rod move with the velocity v with which the rod is moving. As charge is moving in a magnetic field, the magnetic field exerts an average force $F_b = qv \times B$ on each free electron. They move in the direction of force and negative charge gets accumulated at one end and positive charge gets accumulated at the other end. This charge exerts a force on the free electrons when the force exerted by the magnetic field and the electric field due to charges at the two ends are equal, then there is no resultant force on free electrons.

The potential difference between the ends is then vBl where l is the length of the rod. It is the magnetic force on the moving free electrons that maintains the potential difference $V = vBl$ and hence produces an emf

$$\mathcal{E} = vBl.$$

Induced electric field

If the magnetic field changes with time, it is found that induced current starts in closed loop. But as electrons in the conductor are not moving (random motion of electrons is disregarded) magnetic field cannot exert force. These electrons at rest may be forced to move only by an electric field and hence the conclusion that an electric field appears at $t = 0$. The presence of a conducting loop is not necessary to have an induced electric field. As long as magnetic field (B) is changing the induced electric field is present.

The electric field which is responsible for the current is produced by the changing magnetic field and is called the induced electric field. This electric field is nonelectrostatic and nonconservative in nature. The lines of induced electric field are closed curve. There are no starting and terminating points of the field lines.

If E be the induced electric field, the force on a charge placed in the field is qE .

The work done per unit charge as the charge moves through dl is $\int E \cdot dl$

The emf developed in the loop is therefore

$$\epsilon = \int E \cdot dl$$

According to Faraday's law $\epsilon = -d\Phi/dt = \int E \cdot dl$

Eddy Current

When a solid plate of metal is moving in a region having a magnetic field, current may be induced some circular paths in the surface of the metal. There is no definite conducting loop but the system itself results in locating some loops and current termed as eddy current flows through it. The eddy current flow results in thermal energy and this thermal energy comes at the cost of the kinetic energy of the plate and the plate slows down. This slowing down is called electromagnetic damping. To reduce it, slots are cut in the plate and such an arrangement reduces possible paths of the eddy current considerably.

Self induction

When a current is established in a closed conducting loop, it produces a magnetic field. This magnetic field has its flux through an area bounded by the loop. If the current changes with time, the flux through the loop changes and hence an emf is induced in the loop. This process is called self induction as the emf is induced in the loop by changing the current in the same loop.

$$\text{Self induced emf} = -L di/dt$$

Where L is called self-inductance of the loop and is a constant depending on the geometrical construction of the loop.

Self inductance of a long solenoid

Self inductance of a long solenoid is

$$L = \mu_0 n^2 \pi r^2 l$$

Where L = Self inductance

n = number of turns per unit length of the solenoid

r = radius of the solenoid

l = length of the solenoid

So the self-inductance depends only on geometrical factors.

A coil or a solenoid made from a thick wire has negligible resistance but a considerable self-inductance. Such an element is called an ideal inductor and is indicated by a coil symbol.

The self induced emf in a coil opposes the change in the current that has induced it in accordance with the Lenz's law. If the current in the coil is increasing, induced current will be opposite to the original current. If the current in the coil is decreasing, the induced current will be along the original current trying to stop its decrease.

Growth of Current in an LR circuit

LR circuit has a resistance and an inductance.

Applied emf is $\mathcal{E}$

Self induced emf is $-L(di/dt)$

Therefore according to Kirchhoff's loop law $\mathcal{E} - L(di/dt) = Ri$

That means $L(di/dt) = \mathcal{E} - Ri$

By solving the differential equation and substituting the initial conditions that at $t = 0$, $i = 0$ we get the expression for i ,

$$i = i_0(1 - e^{-tR/L})$$

$$= i_0(1 - e^{-t/\tau})$$

where

i = current in the circuit at time t

$$i_0 = \mathcal{E}/R$$

$\mathcal{E}$ = applied emf

R = resistance of the circuit

L = inductance of the circuit

Writing $L/R = \tau$,

$$i = i_0(1 - e^{-t/\tau})$$

$\tau = L/R$ = time constant of the LR circuit

L/R has dimensions of time and is called the time constant of the LR circuit.

The current in the circuit gradually rises from $t = 0$ and attains the maximum value i_0 after a long time.

At $t = \tau$, the current is

$$i = i_0(1 - 1/e) = 0.63 i_0.$$

The time constant indicates how fast will the current grow. If the time constant is small, the growth happens quickly or it steep. While in principle, it may take infinite time for the current to attain its maximum value, in practice in a small number of time constants the current reaches almost its maximum value.

Decay of Current in an LR circuit

By a special arrangement after the current stabilizes in a LR circuit, the battery is disconnected and the circuit is completed without the battery. Hence the current which at time $t = 0$; is i_0 starts decreasing with time. Now only induced emf is in the circuit.

Hence $-L(di/dt) = Ri$

Solving the differential equation we get

$$i = i_0(1 - e^{-tR/L})$$

$$= i_0(1 - e^{-t/\tau}) \text{ where}$$

$\tau = L/R = \text{time constant of the LR circuit}$

Current does not fall to zero immediately, it gradually decreases.

At $t = \tau$,

$$i = i_0/e = 0.37 i_0$$

The current reduces to 37% of the initial current in one time constant. If the time constant is small, the fall or decay will be steep.

Energy stored in an inductor

In a capacitor, when it is charged, electric field builds up between its plates and energy is stored in it. Similarly in the case of inductor, due to flow of current through it, magnetic field builds up in it and magnetic energy is stored in it.

The energy stored in the inductor carrying a current i , is

$$U = \frac{1}{2} Li^2$$

Energy density in magnetic field

$$\text{For a solenoid } L = \mu_0 n^2 \pi r^2 l$$

The magnetic energy therefore is

$$U = \frac{1}{2} Li^2 = \mu_0 n^2 \pi r^2 l i^2$$

For the solenoid with radius r , length l and turns n per unit length, carrying current i , magnetic field within it is

$$B = \mu_0 n i$$

Hence U can be written as $B^2 V / 2\mu_0$

$$V = \text{the volume enclosed by the solenoid} = \pi r^2 l$$

As the field is uniform throughout the volume of the solenoid and zero outside, the energy per unit volume, that is the energy density is

$$u = U/V = B^2 / 2\mu_0$$

Mutual Induction

Mutual induction is induction is due to two closed circuits, with current flowing in one circuit. The current flowing one circuit produces a magnetic field and this field has a flux through the area bounded by the other circuit.

We can write $\Phi = M i$

Where M is a constant and it is called mutual inductance of the pair of circuits, as current in either of the circuits will give the same flux. If the current i in one circuit changes with time, the flux also changes and an emf is induced in the second circuit. This phenomenon is called mutual induction.

$$\mathcal{E} = -d\Phi/dt = -M(di/dt)$$

Induction Coil

An induction coil is used to produce a large emf from a source of low emf.

Ruhmkorff's induction coil is described in the book. There is a make or break arrangement so that current increases in the primary coil and then suddenly decreases. The change in flux due to the change in current in the primary coil induces a large emf in the secondary coil which is in the coaxial position with the primary coil which itself is wound on a soft core.

From 12 V, emf of the order 50,000 V which can operate a discharge tube can be generated with this arrangement.

There is a capacitor in the primary coil circuit to avoid the sparks when current suddenly drops to zero in the primary circuit.

Formula

1. Faraday's law of electromagnetic induction

$$\epsilon = -d\Phi/dt \dots (1)$$

Where

ϵ = emf produced

$\Phi = \int \mathbf{B} \cdot d\mathbf{S}$ = the flux of the magnetic field through the area.

$$2. i = \epsilon/R = -(1/R) d\Phi/dt \dots (2)$$

where i = current in the circuit

R = resistance of the circuit

$$3. \epsilon = vBl$$

Where

ϵ = emf produced

v = velocity of the conductor

B = magnetic field in which the conductor is moving

l = length of the conductor

4. Induced electric field

$$\oint \mathbf{E} \cdot d\mathbf{l} = -d\Phi/dt \dots (4)$$

Where

E = induced electric field due to magnetic field B

5. Self induction

Magnetic field through the area bounded by a current-carrying loop is proportional to the current flowing through it.

$$\Phi = Li \dots (5)$$

Where

$\Phi = \int \mathbf{B} \cdot d\mathbf{S}$ = the flux of the magnetic field through the area.

L = is a constant called the self-inductance of the loop.

i = current through the loop.

6. Self induced EMF

$$\mathcal{E} = -d\Phi/dt = -L di/dt \dots (6)$$

7. Self inductance of a long solenoid

$$L = \mu_0 n^2 \pi r^2 l \dots (7)$$

8. Growth of current through an LR circuit

$$i = i_0(1 - e^{-tR/L}) \dots (8)$$

$$= i_0(1 - e^{-t/\tau}) \dots (9)$$

where

i = current in the circuit at time t

$$i_0 = \mathcal{E}/R$$

$\mathcal{E}$ = applied emf

R = resistance of the circuit

L = inductance of the circuit

$\tau = L/R$ = time constant of the LR circuit

10. Decay of current in a LR circuit

$$i = i_0(1 - e^{-tR/L}) \dots (10)$$

$$= i_0(1 - e^{-t/\tau}) \dots (11)$$

i = current in the circuit at time t

i_0 = current in the circuit at time $t = 0$

R = resistance of the circuit

L = inductance of the circuit

$\tau = L/R$ = time constant of the LR circuit

12. Energy stored in an inductor

$$U = \frac{1}{2} Li^2 \dots (12)$$

13. Energy density

$$u = U/V = B^2/2\mu_0$$

14. Mutual induction

$$\Phi = Mi \dots (14)$$

Where

M = constant called mutual inductance of the given pair of circuits

$$\mathcal{E} = -M di/dt \dots (15)$$