Quadratic Equations

THEORY

An equation of the from

$ax^2 + bx + c = 0$ (1)

Where a, b, c are all real and a is not equal to 0, is a quadratic equation. Then,

$D = (b^2 – 4ac)$ is the discriminant of the quadratic equation.
If $D < 0$ (i.e. the discriminant is negative) then the equation has no real roots.
If $D < 0$ (i.e. the discriminant is positive) then the equation has two distance roots, namely,

$x_1 = (- b +\sqrt{D})/2a$, and $x_2 = (-b - \sqrt{D})/2a$

and then $ax^2 + bx + c = a (x – x_1) (x – x_2)$ (2)

If $D = 0$, then the quadratic equation has equal roots given by

$x_1 = x_2 = -b/2a$

And then $ax^2 + bx + c = a (x – x_1)^2$ (3)

To represent the quadratic $ax^2 + bx + c$ in from (2) or (3) is to expand it into linear factors.

Properties of Quadratic Equations and Their Roots

i. If D is a perfect square then the roots are rational and in it is not a perfect square then roots are irrational.
ii. In the case of imaginary roots ($D < 0$) and if $p + iq$ is one root of the quadratic equation, then the other must be the conjugate $p – iq$ and vice versa (where p and q are real and $i = \sqrt{-1}$)
iii. If $p + \sqrt{q}$ is one root of a quadratic equation, then the other must be the conjugate $p - \sqrt{q}$ and vice versa. (where p is rational and $\sqrt{q}$ is a surd).
iv. If $a = 1$, $b, c \in I$ and the roots are national numbers, then the root must be an integer.
v. If a quadratic equation in x has more than two roots, then it is an identity in x.