

Waves optics

Wave front

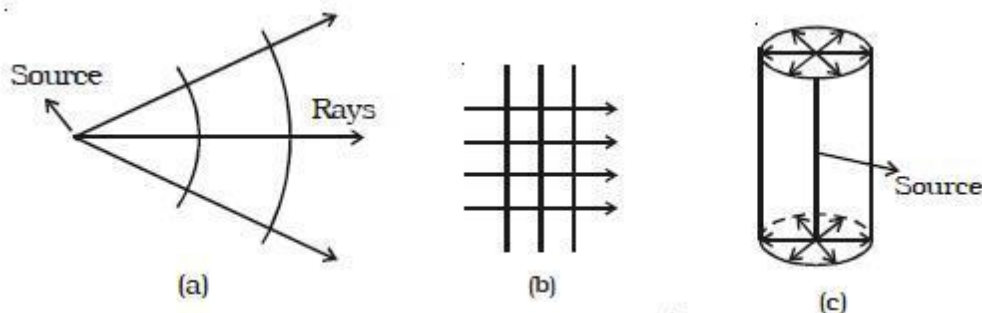
The wave front at any instant is defined as the locus of all the particles of the medium which are in the same state of vibration.

Or

Aan imaginary surface passing through particles oscillating with same phase is known as wavefront A point source of light at a finite distance in an isotropic medium emits a spherical wave front (Fig a).

A point source of light in an isotropic medium at infinite distance will give rise to plane wavefront (Fig. b).

A linear source of light such as a slit illuminated by a lamp, will give rise to cylindrical wavefront (Fig c).



HUYGENS PRINCIPLE

Huygen's principle states that,

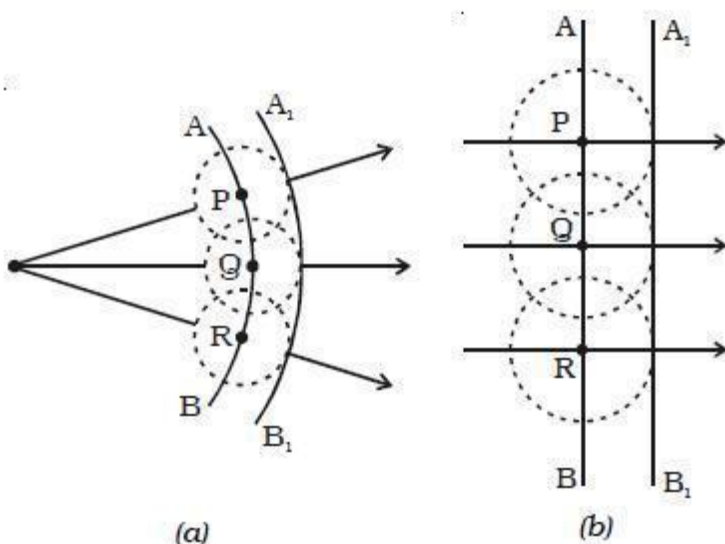
- (i) every point on a given wave front may be considered as a source of secondary wavelets which spread out with the speed of light in that medium and
- (ii) the new wavefront is the forward envelope of the secondary wavelets at that instant

Huygen's construction for a spherical and plane wavefront :

Huygen's construction for a spherical and plane wavefront is shown in Fig.a. Let AB represent a given wavefront at a time $t = 0$. According to Huygen's principle, every point on AB acts as a source of secondary wavelets which travel with the speed of light c . To find the position of the wave front after a time t , circles are drawn with points P, Q, R ... etc as centres on AB and radii equal to ct .

These are the traces of secondary wavelets. The arc A_1B_1 drawn as a forward envelope of the small circles is the new wavefront at that instant.

If the source of light is at a large distance, we obtain a plane wave front $A_1 B_1$ as shown in Fig b.



Refraction of a plane wavefront at a plane surface

Let XY be a plane refracting surface separating two media 1 and 2 of refractive indices μ_1 and μ_2 (Fig). The velocities of light in these two media are respectively v_1 and v_2 .

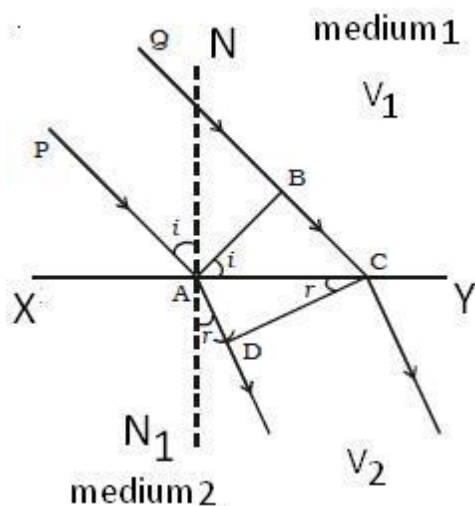
Consider a plane wave front AB incident on the refracting surface at A. PA and QBC are perpendiculars drawn to AB at A and B respectively. Hence they represent incident rays. NAN_1 is the normal drawn to the surface. The wave front and the surface are perpendicular to the plane of the paper.

According to Huygen's principle each point on the wave front act as the source of secondary wavelet. By the time, the secondary wavelets from B, reaches C, the secondary wavelets from the point A would travel a distance $AD = v_2t$, where t is the time taken by the wavelets to travel the distance BC.

$$\therefore BC = C_1t \text{ and } AD = C_2t$$

Taking A as centre and C_2t as radius an arc is drawn in the second medium. From C a tangent CD is drawn to this arc.

Therefore CD is the refracted plane wavefront and AD is the refracted ray



Laws of refraction

- i. The incident wave front AB, the refracted wave front CD and the refracting surface XY all lie in the same plane.

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- ii. From figure for ΔABC and ΔACD

$$\sin i / \sin r = BC/AC / AD/AC = BC/AD = v_1 t / v_2 t = v_1 / v_2 = n_{21}$$

Constant n_{21} in above equation is known as refractive index of medium 2 with respect to medium also represented as ${}_{1\mu 2}$

This is Snell's law of refraction

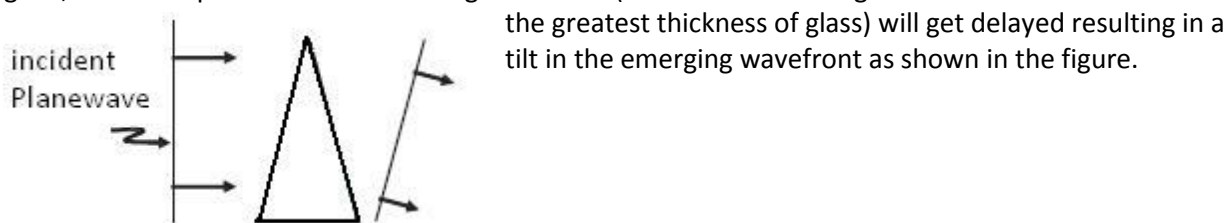
Further, if λ_1 and λ_2 denote the wavelengths of light in medium 1 and medium 2, respectively and if the distance BC is equal to λ_1 then the distance AE will be equal to λ_2 (because if the crest from B has reached C in time τ , then the crest from A should have also reached E in time τ); thus

$$\lambda_1 / \lambda_2 = BC / AE = v_1 / v_2$$

The above equation implies that when a wave gets refracted into a denser medium ($v_1 > v_2$) the wavelength and the speed of propagation decrease but the frequency f ($= v/\lambda$) remains the same.

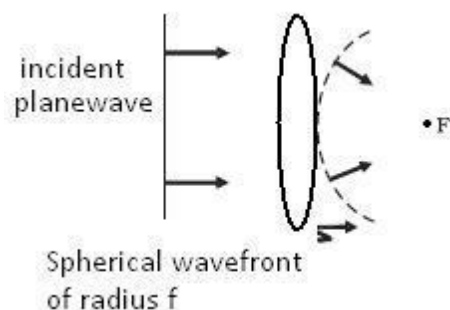
Refraction of a plane wave by a thin prism

We consider a plane wave passing through a thin prism. Clearly, since the speed of light waves is less in glass, the lower portion of the incoming wavefront (which travels through



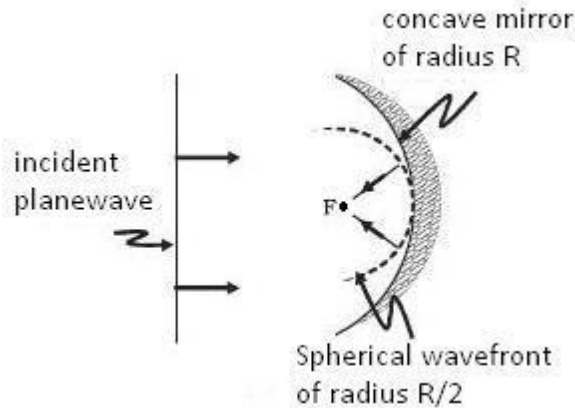
(b) a convex lens.

We consider a plane wave incident on a thin convex lens; the central part of the incident plane wave traverses the thickest portion of the lens and is delayed the most. The emerging wavefront has a depression at the centre and therefore the wavefront becomes spherical and converges to the point F which is known as the focus.



(c) Reflection of a plane wave by a concave mirror

A plane wave is incident on a concave mirror and on reflection we have a spherical wave converging to the focal point F .

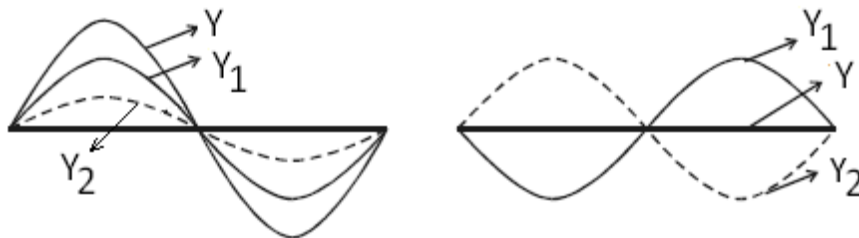


Coherent and incoherent sources

Two sources are said to be coherent if they emit light waves of the same wave length and start with same phase or have a constant phase difference. Two independent monochromatic sources, emit waves of same wave length. But the waves are not in phase. So they are not coherent. This is because, atoms cannot emit light waves in same phase and these sources are said to be incoherent sources.

Superposition principle

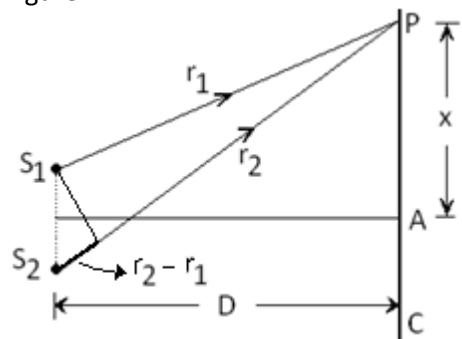
When two or more waves simultaneously pass through the same medium, each wave acts on every particle of the medium, as if the other waves are not present. The resultant displacement of any particle is the vector addition of the displacements due to the individual waves. This is known as principle of superposition. If Y_1 and Y_2 represent the individual displacement then the resultant displacement is given by $Y = Y_1 + Y_2$



Thus, superposition principle describes a situation when more than one waves superpose (i.e. interfere) at a point. "The effect produced by superposition of two or more wave is called interference".

Interference due to two waves

Suppose two harmonic waves having initial phase ϕ_1 and ϕ_2 are emitted from two point like sources S_1 and S_2 respectively. They superimpose simultaneously (i.e. at the same time t) at a point P as shown in figure.



Visible perception of light is produced only by electric field, and therefore, in the present case we write light waves produced by source S_1 and S_2 in terms of electric fields (e) only. Due to S_1 source,

$$e_1 \rightarrow = E_1 \rightarrow \sin(\omega_1 t - k_1 r_1 + \phi_1)$$

And due to source S_2

$$e_2 \rightarrow = E_2 \rightarrow \sin(\omega_2 t - k_2 r_2 + \phi_2)$$

Here E_1 and E_2 represent amplitude of electric fields, ω_1 and ω_2 denotes angular frequencies of waves, and k_1 and k_2 are wave vectors.

Let $\delta_1 = \omega_1 t - k_1 r_1 + \phi_1$ and $\delta_2 = \omega_2 t - k_2 r_2 + \phi_2$

Then $e_1 = E_1 \sin \delta_1$ and $e_2 = E_2 \sin \delta_2$

Now according to principle of superposition

$$e = e_1 + e_2$$

magnitude of resultant vector e

$$e^2 = e_1^2 + e_2^2 + 2e_1 e_2 \cos(\delta_1 - \delta_2)$$

If at a instant of time E_1 and E_2 amplitude of waves then, resultant amplitude E is

$$E^2 = E_1^2 + E_2^2 + 2E_1 E_2 \cos(\delta_1 - \delta_2)$$

The average intensity of light is proportional to square of amplitude $I \propto E^2$ thus equation becomes

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \langle \cos(\delta_1 - \delta_2) \rangle$$

In above equation I_1 and I_2 are the average intensities due to each wave. They are independent of time.

The last term in above equation is known as interference term which depends on time

Now

$$\langle \cos(\delta_1 - \delta_2) \rangle = \frac{1}{T} \int_{t_0}^{t_0+T} \cos(\delta_1 - \delta_2) dt$$

Here t is period of electric field oscillation. On substituting value of δ_2 and δ_1 in above equation

$$\langle \cos(\delta_1 - \delta_2) \rangle = \frac{1}{T} \int_{t_0}^{t_0+T} \cos\{(\omega_1 t - \omega_2 t) + (k_2 r_2 - k_1 r_1) + (\phi_2 - \phi_1)\} dt \quad \text{---(1)}$$

Case I: Incoherent sources

If angular frequency of both source is not same thus $\cos(\delta_1 - \delta_2)$ is time dependent and average value is zero. Thus superposed two waves produce the average intensity $I_1 + I_2$ at point P

Case II: Coherent sources: For sources to be coherent there angular frequency should be same thus $\omega_1 = \omega_2 = \omega$ (say) Also since both waves are travelling in same medium there speed will be also same thus wave length is same thus $k_1 = k_2 = k$ (say) for sake of simplicity we will consider $\phi_2 = \phi_1$. From equation (1) ignoring negative sign of $\cos$

$$\langle \cos(\delta_1 - \delta_2) \rangle = \frac{1}{T} \int_0^T \cos\{k(r_2 - r_1)\} dt$$

$$\langle \cos(\delta_1 - \delta_2) \rangle = \frac{1}{T} \cos\{k(r_2 - r_1)\} \int_0^T dt$$

$$\langle \cos(\delta_1 - \delta_2) \rangle = \cos\{k(r_2 - r_1)\} \quad \text{--- eq(2)}$$

Further we will assume that amplitude of both waves is equal $I_1 = I_2 = I'$ then

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From equation (1) and eq(2) we get

$$I = I' + I' + 2\sqrt{I'I'}\cos(r_2 - r_1)$$

$$I = 2I' + 2I'\cos(r_2 - r_1)$$

$$I = 2I' \{1 + \cos(r_2 - r_1)\}$$

$$I = 2I' [2\cos^2 \{ (r_2 - r_1)/2 \}]$$

[Use trigonometric identity $\cos^2 u = 1 + \cos 2u/2$]

$$I = 4I'\cos^2 \{ (r_2 - r_1)/2 \}$$

Here $r_2 - r_1 = \delta$ is known as the path difference between superposing waves

$$I = 4I'\cos^2 \{ \delta/2 \}$$

Special Cases

Case I : Constructive Interference For $I = 4I' = I_0$ maximum intensity of light

term $\cos^2 \{ k(r_2 - r_1)/2 \}$ Should be equal to one, It is possible if

$$k\delta/2 = n\pi$$

$$2\pi\delta/2\lambda = n\pi \because k = 2\pi/\lambda$$

$$\delta = n\lambda$$

Here $n = 0, 1, 2, 3, \dots$

"If the path difference between superposing waves is $n\lambda$ ($n = 0, 1, 2, 3, \dots$) intensity at a superposing point is maximum. Such interference is called constructive interference" From equation

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\delta_1 - \delta_2)$$

For constructive interference $\cos(\delta_2 - \delta_1) = 1$ thus

$$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

Or

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

$$I_{max} \propto (A_1 + A_2)^2$$

Case II: Destructive Interference

For intensity $I = 0$

term $\cos^2 \{ k(\delta)/2 \}$ Should be equal to zero, It is possible if

$$k\delta/2 = \pi/2, 3\pi/2, 5\pi/2, \dots$$

Or

$$k\delta/2 = (2n - 1)\pi/2$$

As $k = 2\pi/\lambda$

$$2\pi\delta/2\lambda = (2n - 1)\pi/2$$

$$\delta = (2n - 1)\lambda/2$$

Here $n = 1, 2, 3, 4, \dots$

"If phase difference between superposing waves is $(2n-1)\pi$ intensity at a superposing point is minimum. This interference is called destructive interference"

"If path difference between superposing wave is $(2n-1)(\lambda/2)$ intensity at superposed point is minimum. Such interference is known as destructive interference"

Here $n = 1, 2, 3, 4, \dots$ From equation

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos(\delta_1 - \delta_2)$$

For destructive interference $\cos(\delta_1 - \delta_2) = -1$ thus

$$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

Or

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$$I_{min} \propto (A_1 - A_2)^2$$

Condition for sustained interference

The interference pattern in which the positions of maximum and minimum intensity of light remain fixed with time, is called sustained or permanent interference pattern. The conditions for the formation of sustained interference may be stated as :

- (i) The two sources should be coherent
- (ii) Two sources should be very narrow
- (iii) The sources should lie very close to each other to form distinct and broad fringes

Solved numerical

Q) Two sources of intensity I and $3I$ are used in an interference experiment. Find the intensity at a point where the waves from the two sources superimpose with a phase difference (1) Zero (2) $\pi/2$

Solution:

In case of interference

$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2} \langle \cos(\delta_1 - \delta_2) \rangle$$

$$I' = I_1 + I_2 + 2\sqrt{I_1 I_2} \langle \cos(\delta) \rangle$$

(1) As $\delta = 0$, $\cos\delta = 1$

$$\therefore I' = 3I + I + 2\sqrt{(3I)(I)} \times 1 \quad I' = 4I + 2\sqrt{3I} \quad (2) \text{ As } \delta = \pi/2, \cos\delta = 0$$

$$\therefore I' = 3I + I + 2\sqrt{(3I)(I)} \times 0 \therefore I' = 4I$$

Location of bright and dark fringes on screen (x)

Bright fringes:

By the principle of interference, condition for constructive interference is the path difference = $n\lambda$

$$n\lambda = xd/D$$

$$x = n\lambda D/d$$

By substituting $n = 0, 1, 2, 3, \dots$

If $n = 0$ then we get location central bright fringe

$n=1$, we get then location of first bright fringe

$n=2$, we get then location of second bright fringe

$n=3$, we get then location of third bright fringe... etc

Dark fringes:

By the principle of interference, condition for destructive interference is the path difference

$$\delta = (2n - 1) \lambda/2$$

$$(2n - 1) \lambda/2 = xd/D$$

$$x = (2n - 1)\lambda/2d$$

By substituting $n = 1, 2, 3, \dots$

$n=1$, we get then location of first dark fringe

$n=2$, we get then location of second dark fringe

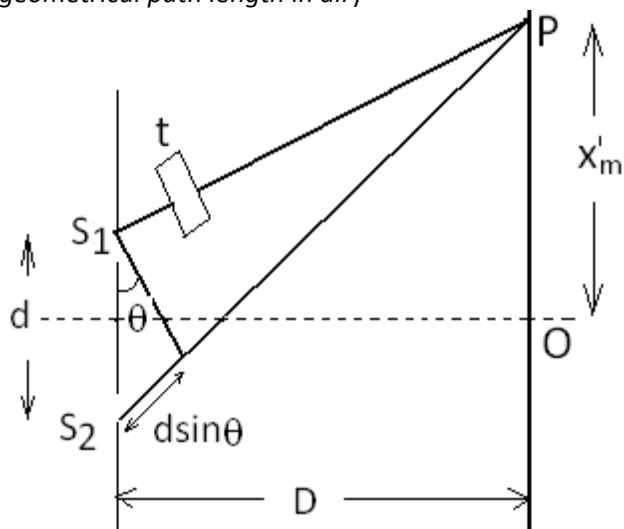
$n=3$, we get then location of third dark fringe... etc

Displacement of fringe pattern

If a thin transparent slab of thickness t and refractive index μ is placed in front of one of sources, for example, in front of S_1 . This changes the path difference because light from S_1 now travels more optical

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path than earlier. (Optical path in medium is equal to the product of refractive index of the medium to geometrical path length in air)



Path difference before placing slab = $S_2P - S_1P = r_2 - r_1 = \delta$

On placing slab path length of thickness t effective path length $S_1P = (r_1 - t) + t\mu$

Thus effective path difference after placing slab $\delta' = r_2 - [(r_1 - t) + t\mu]$

$$\therefore \delta' = (r_2 - r_1) + t(\mu - 1)$$

From the geometry of figure $r_2 - r_1 = d \sin \theta$, since θ is very small

$$r_2 - r_1 = d \tan \theta = dx'_m/D$$

$$\therefore \delta' = dx'_m/D + (\mu - 1)t$$

$$x'_m = n\lambda D/d - (\mu - 1)tD/d$$

In absence of slab, the m th maxima is given by

$$x_m = n\lambda D/d$$

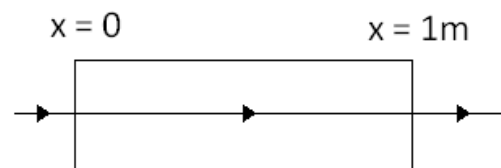
Therefore, the fringe shift is given by

$$x_0 = x_m - x'_m = (\mu - 1)tD/d$$

When a transparent slab is introduced, the fringe pattern shifts in the direction where the slab is placed.

Solved numerical

Q) A ray of light travels through a slab as shown in figure. The refractive index of the material of the slab varies as $\mu = 1.2 + x$, where $0 \leq x \leq 1$ m. What is the equivalent optical path of the glass slab?



Solution

Consider a small geometric path dx then optical path = μdx

Thus optical path $op =$

$$op = \int_0^1 (1.2 + x)dx = [1.2x + x^2/2]_0^1$$

$$op = 1.2 + 1/2 = 3.4/2 = 1.7$$

Distance between two consecutive bright or dark fringe Or Width of fringe , Band width (β)

The distance between any two consecutive bright or dark bands is called bandwidth. The distance between $(n+1)^{\text{th}}$ and n^{th} order consecutive bright fringes from O is given by

$$\beta = x_{n+1} - x_n = (n+1) \lambda D/d - (n) \lambda D/d = \lambda D/d$$

Similarly, it can be proved that the distance between two consecutive dark bands is also equal to $\lambda D/d$

Angular fringe width or angular separation between fringes is

$$\theta = \lambda/d$$

Since bright and dark fringes are of same width, they are equi-spaced on either side of central maximum.

Solved numerical

Q) In Young's double slit experiment, angular width of a fringe formed on a distant screen is 0.10° . The wavelength of the light used is $6000\text{\AA}$. What is the spacing between the slit. If above setup is immersed in liquid it is observed that angular fringe width is decreased by 30% find refractive index of liquid

Solution

Angular fringe width or angular separation between fringes is

$$\theta = \lambda/d$$

$$d = \lambda/\theta = 6000 \times 10^{-10} / 0.1 \times \pi/180 = 3.44 \times 10^{-4} \text{m}$$

(ii) Given that when set up with first light is immersed in liquid angular width decreases by 30% thus wave length of first light in liquid = $4200\text{\AA}$

From formula for refractive index

$$\mu = \lambda_{\text{air}}/\lambda_{\text{liq}} = 6000/4200 = 1.428$$

Condition for obtaining clear and broad interference bands

(i) The screen should be as far away from the source as possible.

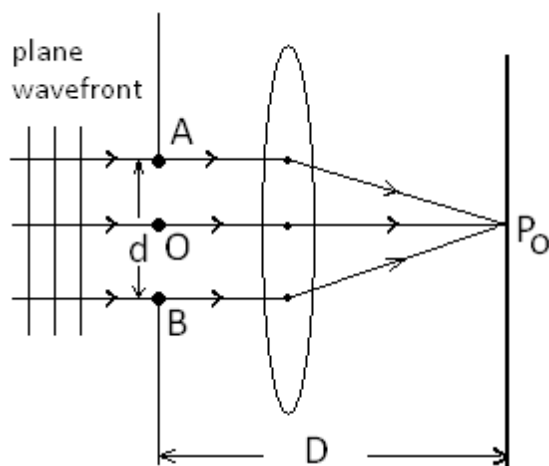
(ii) The wavelength of light used must be larger.

(iii) The two coherent sources must be as close as possible.

Diffraction

When waves encounter obstacles or openings like slits, they bend round the edges. This bending of wave is called diffraction. Diffraction is the effect produced by the limiting part of the wavefront. Smaller is the width of the slit, more will be diffraction for given wavelength. It is also found that if the wavelength and the width of the slit are so changed that ratio (λ/d) remains constant, amount of bending or diffraction does not change. If ratio λ/d is more, then more is the diffraction

Diffraction due to single slit



Central Maxima:

Consider a plane wavefront arrive at a plane of slit, according to Huyen's principle all the point on the slit like AOB acts as secondary source having the same phase and produce secondary waves.

Those waves originated from each points of a slit and diffracted normal to the plane of the slit or we can say in the direction of incident wave will be concentrated at point P_0 by lens. In figure out of a many waves only three rays are shown in figure.

Screen is at focal length of lens .

Ray emitted from A and B are in phase and passes equal distance through air and lens thus they are in phase when get converged at P_0 .

Now ray emitted from O travel less distance in air but more distance in lens, in lens velocity of light gets reduced thus optical path travelled by the ray emitted by O is equal to optical path due to ray A and B..

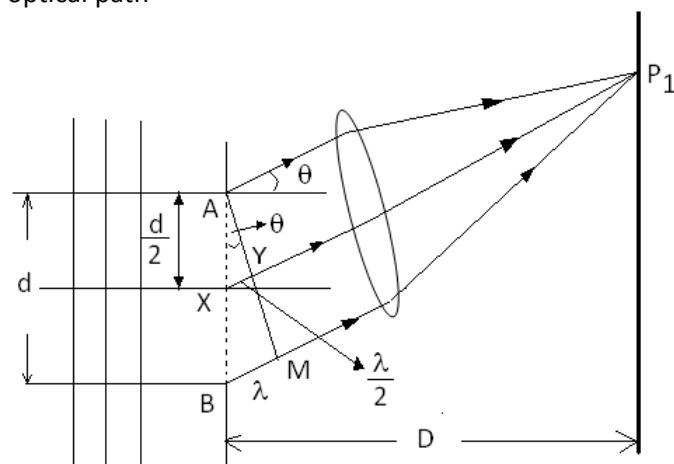
Thus all rays meeting at P_0 are in phase produces central bright fringe.

(Optical path in medium is equal to the product of refractive index of the medium to geometrical path length in air)

First minimum

As shown in figure consider a waves which is diffracted an angle θ with respect to perpendicular bisector XP_0 of the slit. Here, point X is the midpoint of slit AB. Therefore $AX = Xb = d/2$.

Here secondary waves originated from all points A, X, B of slit are through to be divided in two parts Wave from AX and waves from X to B. As per figure, all these waves diffracted at an angle θ are focused at point P_1 of a screen. Draw $AM \perp BL$. It is obvious that all the rays reaching from AM to P_1 have equal optical path



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But rays going from A and X, and reaching to point P_1 have path difference of $\lambda/2$. Let us assume diffraction angle θ is such that $XY = \lambda/2$.

In this situation, waves from A and X will follow the condition of destructive interference at point P_1 and resultant intensity will be zero.

Further, for all points between AX, there exists a point between XB, such that a ray from a point between XB has a path difference of $\lambda/2$ with respect to rays from a point between AX.

Thus in totality, destructive interference will take place at point P_1 .

Point P_1 is known as first minimum.

Condition for minima:

From geometry of figure $d \sin \theta = \lambda$

General equation is

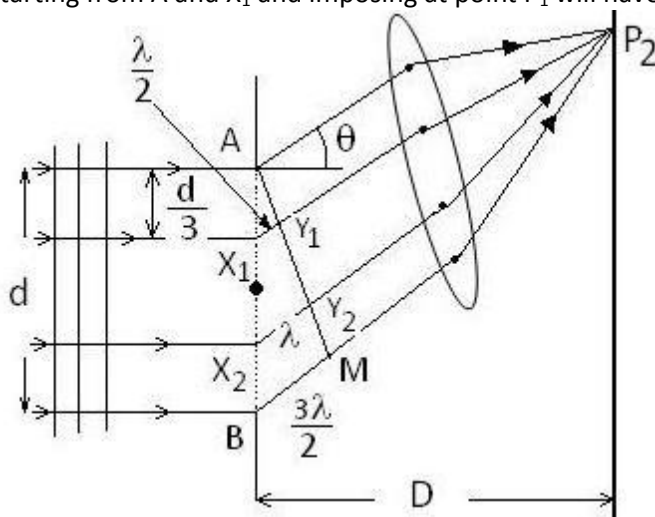
$$d \sin \theta = n \lambda$$

For $n = 1$ we get first minima

$n = 2$ we get second minima

First Maxima:

As shown in figure, suppose slit AB is assumed to be divided into three equal parts AX_1 , X_1X_2 , X_2B . Here $AX_1 = X_1X_2 = X_2B = d/3$. Draw $AM \perp BL$. Wave reaching from AM to P_2 will have equal optical path. Waves starting from A and X_1 and interfering at point P_1 will have path difference X_1Y_1 .



Let us assume that diffraction θ is such that

$$X_1Y_1 = \lambda/2$$

$$X_2Y_2 = \lambda$$

$$BM = 3\lambda/2$$

Since the path difference between waves originating from A and X_1 and superimposed at point P_2 is $\lambda/2$, they interfere destructively. And the intensity at point P_2 due to these waves will be zero.

In the same way, waves from every pair AX_1 and X_1X_2 will have a path difference $\lambda/2$ and the resultant intensity at point P_2 due to them is zero.

However, the intensity of a ray diffracted at an angle θ from section X_2B is not vanishing at point P_2 . Therefore, due to this section of the slit, the intensity at point P_2 will not be zero. And point P_2 will be bright.

Here point P_2 is known as first maximum. It is obvious that the intensity at point P_2 will be far less than the central bright spot.

Condition for minima:

From geometry of figure for first maxima

$$d \sin \theta = 3\lambda/2$$

General formula

$$d \sin \theta = (2n + 1)\lambda/2$$

For $n = 1$ we get first maxima

$n = 2$ we get second maxima

Angular width

For first order minima $d \sin \theta = \lambda$ or

$$\sin \theta = \lambda/d$$

For small angle

$$\theta = \lambda/d$$

Also $\sin \theta = \tan \theta$

From geometry of figure

$$\tan \theta = x/D$$

$$\therefore x/D = \lambda/d$$

$\therefore$ width of central maxima

$$2x = 2\lambda D/d$$

Angular width of central maxima is given by

$$2\theta = 2\lambda/d$$

Comparison between Interference and diffraction

(i) The interference pattern has a number of equally spaced bright and dark bands. The diffraction pattern has a central bright maximum which is twice as wide as the other maxima. The intensity falls as we go to successive maxima away from the centre, on either side.

(ii) We calculate the interference pattern by superposing two waves originating from the two narrow slits. The diffraction pattern is a superposition of a continuous family of waves originating from each point on a single slit.

(iii) For a single slit of width a , the first null of the interference pattern occurs at an angle of λ/a . At the same angle of λ/a , we get a maximum (not a null) for two narrow slits separated by a distance a .

Solved numerical

Q) Angular width of central maximum in diffraction obtained by single slit using light of wavelength $6000\text{\AA}$ is measured. If light of another wavelength is used, the angular width of the central maximum is found to be decreased by 30%. Find (i) The other wavelength (ii) If the experiment is repeated keeping the apparatus in a liquid, the angular width of central maxima decreases by the same amount (30%), find its refractive index

Solution:

(i) Angular fringe width or angular separation between fringes is

$$2\theta = 2\lambda/d$$

For first light

$$\theta_1 = \lambda_1/d$$

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For second light

$$\theta_2 = \lambda_2/d$$
$$\theta_2/\theta_1 = \lambda_2/\lambda_1$$

But θ_2 is 70% of θ_1

That is, $\theta_2 = 0.7 \theta_1$

$$\therefore 0.7 = \lambda_2/\lambda_1$$
$$\lambda_2 = 0.7 \times 6000 = 4200 \text{ \AA}$$

Resolving power of optical instrument

When a beam of light (light waves) from a point like object passes through the objective of an optical instrument, the lens acts like a circular aperture and produces a diffraction pattern instead of sharp point image.

If there are two point objects kept close to each other, their diffraction pattern may overlap. Then it may be difficult to distinguish them as separate.

The criterion to get distinct and separate images of two closely placed point like objects was given by Rayleigh

“The images of two point like objects can be seen as separate if the central maximum in the diffraction pattern of one falls either on the first minimum of the diffraction pattern of the other or it is at greater separation”

For the case of circular aperture diffraction due to lens of diameter D . Rayleigh's criterion is given by

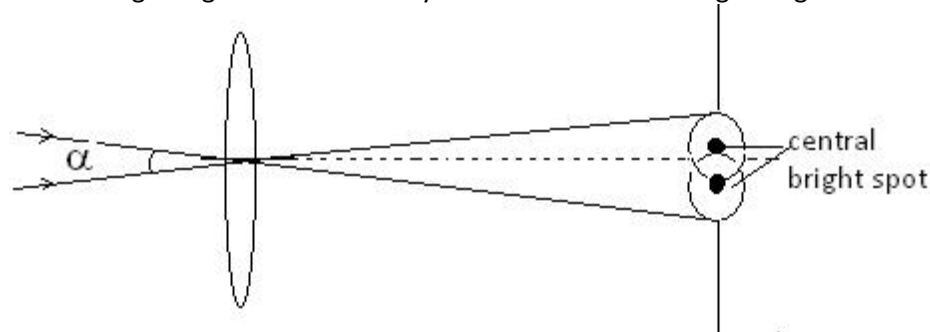
$$\sin \alpha \approx \alpha = 1.22\lambda/D$$

Resolving power of telescope:

Consider a parallel beam of light falling on a convex lens. If the lens is well corrected for aberrations, then beam will get focused to a point.

However, because of diffraction, the beam instead of getting focused to a point gets focused to a spot of finite area. In this case the effects due to diffraction can be taken into account by considering a plane wave incident on a circular aperture followed by a convex lens.

Taking into account the effects due to diffraction, the pattern on the focal plane would consist of a central bright region surrounded by concentric dark and bright rings.



If two stars are very close to each other separated by angle α will be very small and the diffraction pattern of both stars will mingle with each other. In this situation it is difficult to see both the stars distinctly and clearly

“Ability of an optical instrument to produce distinctly separate images of two closely placed objects is

Wave optic

called its resolving power"

It is clear that for optical instruments resolving power depends on angle α . is a minimum angle to see two images distinctly

$$\alpha_{min} = 1.22\lambda/D$$

Here D is diameter of lens and λ is wavelength

Width of the central maxima or radius is given by

$$\alpha_{min} f = 1.22\lambda/D f$$

Here α_{min} is known as angular resolution of the telescope, while its inverse is known as resolving power or geometrical resolution

Thus resolving power of telescope

$$1/\alpha_{min} = D/1.22\lambda$$

Solved numerical

Q) Calculate the useful magnifying power of a telescope of 11cm objective. The limit of resolution of eye is $2'$ and wavelength of light used is $5000\text{\AA}$

Solution

The magnifying power of a telescope is given by

$M = D/d$, where D is diameter of the objective and d is diameter of eye piece

For normal (useful) magnification, diameter of eyepiece should be equal to the diameter of the pupil d_e of the eye. Therefore, useful magnification is

$$M = D/d_e$$

From the equation of limit of resolution of telescope

$$d\theta = 1.22\lambda/D$$

$$d\theta = 1.22 \times 5500 \times 10^{-10} / 11 \times 10^{-2} = 6.1 \times 10^{-6} \text{ rad}$$

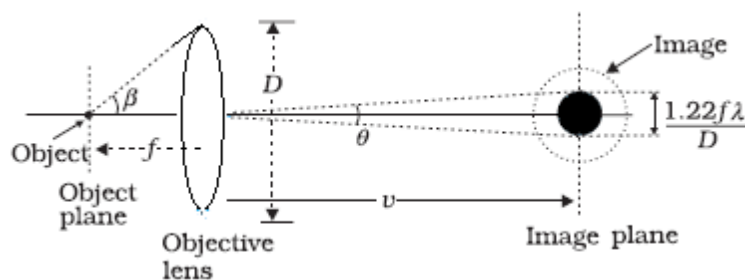
Limit of resolution of eye is given $d\theta' = 2'$

$$d\theta' = 2 \times 3.14 / 60 \times 180^\circ = 5.815 \times 10^{-4} \text{ rad}$$

$\therefore$ Useful magnification

$$d\theta' / d\theta = 5.815 \times 10^{-4} / 6.1 \times 10^{-6} = 95.3$$

Resolving power of microscope:



Let the diameter of lens be D and its focal length be f. As object distance is usually kept greater than that of f. Let the image distance be v. the angular width of central maximum due to the effect of diffraction is ,

$$\theta = 1.22\lambda/D$$

Width of central maximum

$$\theta v = 1.22\lambda/D v$$

Wave optic

If image of two point like objects are at a separation less than $v\theta$, then it will be seen as a mixed single object. It can be proved that a minimum distance (d_m) for which objects can be seen separately is given by

$$d_m = \theta v/m$$

Here m is magnification $m=v/f$ substituting value of m we get d_m

$$d_m = \theta v/v/f = \theta f$$

Substituting value of θ we get

$$d_m = 1.22\lambda/D f$$

From figure $D/2 = f(\tan\beta)$

$D = 2f(\tan\beta)$ substituting value in above equation we get

$$d_m = 1.22\lambda/2f \tan\beta \quad \beta f = 1.22\lambda/2 \tan\beta$$

For small angles $\tan\beta = \sin\beta$

Reciprocal of d_m known as Resolving Power(RP) of microscope

$$RP = 1/d_m = 2\sin\beta/1.22\lambda$$

If some medium with large refractive index (n) is used between object and objective resolving power of microscope increases n times

Formula for resolving power is given by

$$RP = 2n\sin\beta/1.22\lambda$$

Here term $n\sin\beta$ is known as 'Numerical Aperture'. Resolving power is inversely proportional to wavelength.

Polarization

The phenomena of reflection, refraction, interference, diffraction are common to both transverse waves and longitudinal waves. But the transverse nature of light waves is demonstrated only by the phenomenon of polarization.

Unpolarized light

In an ordinary light source like bulb, there are large numbers of atomic emitters. They all emit electromagnetic waves with their Electrical vector E , vibrating randomly in all directions perpendicular to direction of propagation.

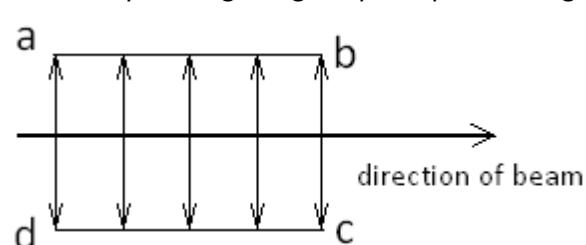
It means that vector E of one wave is not parallel to Vector E of another wave. Wave emitted by different atom of source propagate in same direction constitute beam of light.

If such beam is assumed to be coming out of paper, light vectors (E) of its waves will be found in all random direction in a plane of paper. Such light is called Unpolarized light.

"In a beam of light, if the oscillations of E vectors are in all direction in a plane perpendicular to the direction of propagation, then the light is called unpolarized light"

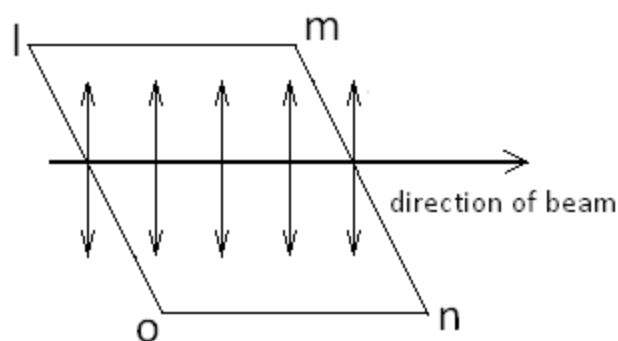
Polarized light

If in beam of light all electric vector (E) are coplanar and parallel to each other is plane polarized light
Process by which getting the plane polarized light from unpolarized light is called



polarization

"The plane containing the direction of the beam and the direction of oscillation of E vectors is called the plane of oscillation. In figure $abcd$ is the plane of oscillation



“A plane perpendicular to the plane of oscillation and passing through the beam of light is called the plane of polarization”

In figure Imno is the plane of polarization

When light passes through tourmaline crystal, freely transmit the light components which are polarized to a definite direction. While crystal absorbs light strongly whose polarization is perpendicular to this definite direction. Thus emergent beam of light only coplanar and parallel E vectors are found. This definite direction in a crystal is known as an optic axis

Malus' Law

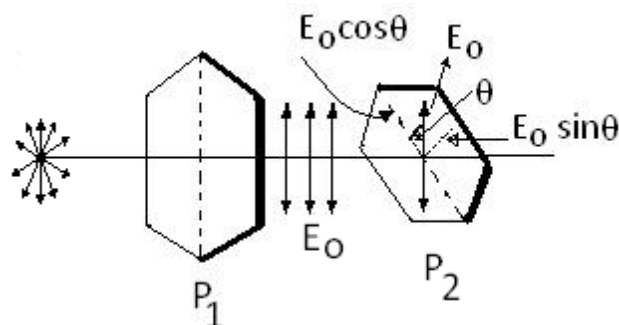
If the light from an ordinary source (like a sodium lamp) passes through a polaroid sheet P_1 , it is observed that its intensity is reduced by half. Rotating P_1 has no effect on the transmitted beam and transmitted intensity remains constant.

Now, let an identical piece of polaroid P_2 be placed before P_1 . On rotating P_2 has a dramatic effect on the light coming from P_2 .

In one position, the intensity transmitted by P_2 followed by P_1 is nearly zero. When turned by 90° from this position, P_1 transmits nearly the full intensity emerging from P_2

An optic axis of plate P_2 makes an angle of θ with that of the plate P_1 . In this situation vector E emerging from plate P_1 (E_0) makes angle θ with an optic axis of plate B. therefore we can resolve them into two components

- (1) $E_0 \cos \theta$ parallel to the optic axis of plate P_2 and
- (2) $E_0 \sin \theta$ perpendicular to the optic axis of plate P_2



Thus, only $E_0 \cos \theta$ components will emerge out of plate P_2 , while perpendicular components are absorbed. Since intensity is proportional to the square of amplitude, intensity of light incident on plate P_2 is

$$I \propto E_0^2 \cos^2 \theta$$

$$\therefore I/I_0 = \cos^2 \theta$$

$$\therefore I = I_0 \cos^2 \theta$$

This equation is known as Malus Law. It is obvious from above equation that if plate P_2 is completely rotated, twice the intensity of emerging light is zero, corresponding to $\theta = \pi/2$ and $3\pi/2$ and twice it become maximum corresponding to $\theta = 0$ and $\theta = \pi$.

This procedure will help us to verify whether the given light is polarized or not. Since plate P_2 is used to analyze a state of polarization of incident light, it is known as Analyzer.

Solved numerical

Q) A ray of light travelling in water is incident on a glass plate immersed in it. What the angle of incident is 51° the reflected ray is totally plane polarized. Find the refractive index of glass. Refractive index of water is 1.33

Solution:

Angle of incidence $\theta_P = 51^\circ$

Since at this incidence angle, reflected ray is totally plane polarized, using Brewster's law refractive index of glass w.r.t. water is

$$n' = \tan \theta_p = \tan 51 = 1.235$$

But refractive index $n' =$

$$n' = RI. \text{ glass}(n_g) / R. i. \text{ of water}(n_w)$$

$$n_g = n' n_w = 1.235 \times 1.33 = 1.64$$

Brewster's law

Sir David Brewster conducted a series of experiments with different reflectors and found a simple relation between the angle of polarization and the refractive index of the medium. It has been observed experimentally that the reflected and refracted rays are at right angles to each other, when the light is incident at polarizing angle.

From Fig, $i_p + 90^\circ + r = 180^\circ$

$$r = 90^\circ - i_p$$

From Snell's law,

$$\sin i_p / \sin r = \mu$$

where μ is the refractive index of the medium (glass)

Substituting for r , we get

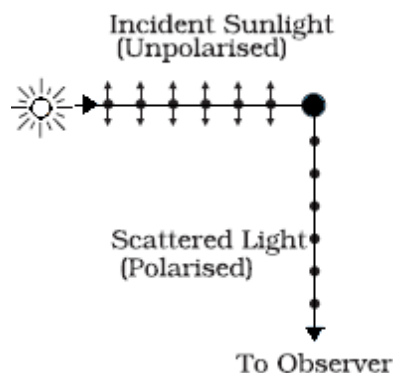
$$\sin i_p / \sin(90^\circ - i_p) = \sin i_p / \cos i_p = \mu$$

$$\therefore \tan i_p = \mu$$

Tangent of polarizing angle is numerically equal to refractive index of medium

Polarisation by scattering

The light from a clear blue portion of the sky shows a rise and fall of intensity when viewed through a Polaroid which is rotated. This is nothing but sunlight, which has changed its direction (having been scattered) on encountering the molecules of the earth's atmosphere



As shown in figure, the incident sunlight is unpolarised. The dots stand for polarisation perpendicular to the plane of the figure. The double arrows show polarisation in the plane of the figure. There is no phase relation between these two in unpolarised light.

Under the influence of the electric field of the incident wave the electrons in the molecules acquire components of motion in both these directions.

We have drawn an observer looking at 90° to the direction of the sun. Clearly, charges accelerating parallel to the double arrows do not radiate energy towards this observer since their acceleration has no transverse component. The radiation scattered by the molecule is therefore represented by dots. It is polarized perpendicular to the plane of the figure. This explains the polarization of scattered light from the sky.

Fresnel distance, ray optics is a limiting case of wave optics

Fresnel distance is that distance from the slit at which the spreading of light due to diffraction becomes equal to the size of the slit. It is generally denoted by Z_F

We know that the first secondary minimum is formed at an angle θ_1 such that

$$\theta_1 = \lambda/d$$

After travelling a distance D , the width acquired by the beam due to diffraction is $D\lambda/d$. At Fresnel distance Z_F

$$\begin{aligned} Z_F \lambda / d &= d \\ Z_F &= d^2 / \lambda \end{aligned}$$

If the distance D between the slit and the screen is less than Fresnel distance Z_F then the diffraction effects may be regarded as absent. So, ray optics may be regarded as limiting case of wave optics

Solved Numerical

Light of wave length 600nm is incident on an aperture of size 2mm. calculate the distance upto which the ray of light can travel such that its spread is less than the size of the aperture

Solution

$$Z_F = d^2 / \lambda = (2 \times 10^{-3})^2 / 600 \times 10^{-9} = 6.67 \text{ m}$$

Doppler effect for light

If there is no medium and the source moves away from the observer, then later wavefronts have to travel a greater distance to reach the observer and hence take a longer time. The time taken between the arrival of two successive wavefronts is hence longer at the observer than it is at the source.

Thus, when the source moves away from the observer the frequency as measured by the source will be smaller. This is known as the *Doppler effect*.

Astronomers call the increase in wavelength due to doppler effect as red shift since a wavelength in the middle of the visible region of the spectrum moves towards the red end of the spectrum.

When waves are received from a source moving towards the observer, there is an apparent decrease in wavelength, this is referred to as *blue shift*.

For velocities small compared to the speed of light, we can use the same formulae which we use for sound waves. The fractional change in frequency $\Delta v/v$ is given by $-V_{\text{radial}}/c$, where V_{radial} is the component of the source velocity along the line joining the observer to the source relative to the observer; V_{radial} is considered positive when the source moves away from the observer. Thus, the Doppler shift can be expressed as:

$$\Delta v/v = -V_{\text{radial}}/c$$

Solved numerical

Q) Certain characteristic wavelengths of the light from a galaxy in the constellation Virgo are observed to be increased in wave length, as compared with terrestrial sources, by 0.4%. What is the radial speed of this galaxy with respect to the earth? Is it approaching or receding?

Solution

From formula

$$\Delta v/v = -V_{\text{radial}}/c$$

We know that

$$\Delta v/v = -\Delta\lambda/\lambda$$

Thus

$$\Delta\lambda/\lambda = V_{\text{radial}}/c$$

Given : $\Delta\lambda/\lambda = 0.004$

$$V_{\text{radial}} = \Delta\lambda/\lambda \cdot c$$

$$V_{\text{radial}} = 0.004 \times 3 \times 10^8 = 1.2 \times 10^6 \text{ ms}^{-1}$$

Since v_{radial} is positive therefore galaxy is receding