PHYSICS NOTES

Motion In One Dimension

Particle

A particle is ideally just a piece or a quantity of matter, having practically no linear dimensions but only a position. In practice it is difficult to get such particle, but in certain circumstances an object can be treated as particle. Such circumstances are

(i) All the particles of solid body performing linear motion cover the same distance in the same time. Hence motion of such a body can be described in terms of the motion of its constituent particle

(ii) If the distance between two objects is very large as compared to their dimensions, these objects can be treated as particles. For example, while calculating the gravitational force between Sun and Earth, both of them can be considered as particles.

Frame of reference

A "frame of reference" is just a set of coordinates: something you use to measure the things that matter in Newtonian problems, that is to say, positions and velocities, so we also need a clock. Or A place and situation from where an observer takes his observation is called frame of reference. A point in space is specified by its three coordinates (x, y, z) and an "event" like, say, a little explosion, by a place and time: (x, y, z, t).

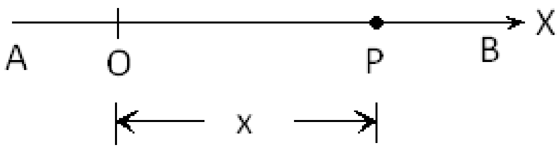
An inertial frame is defined as one in which Newton's law of inertia holds—that is, anybody which isn't being acted on by an outside force stays at rest if it is initially at rest, or continues to move at a constant velocity if that's what it was doing to begin with. Example of inertial frame of reference is observer on Earth for all motion on surface of earth. Car moving with constant velocity An example of a non-inertial frame is a rotating frame, such as a accelerating car,

Rest and Motion

When a body does not change its position with respect to time with respect to frame of reference, then it is said to be at rest. Revolving earth Motion is the change of position of an object with respect to time. To study the motion of the object, one has to study the change in position (x,y,z coordinates) of the object with respect to the surroundings. It may be noted that the position of the object changes even due to the change in one, two or all the three coordinates of the position of the objects with respect to time. Thus motion can be classified into three types:

(i) Motion in one dimension
Motion of an object is said to be one dimensional, if only one of the three coordinates specifying the position of the object changes with respect to time.
Example : An ant moving in a straight line, running athlete, etc. Consider a particle moving on a straight line AB. For the analysis of motion we take origin. O at any point on the line and x-axis along the line. Generally we take origin

at the point from where particle starts its motion and rightward direction as positive xdirection. At any moment if article is at P then its position is given by OP = x



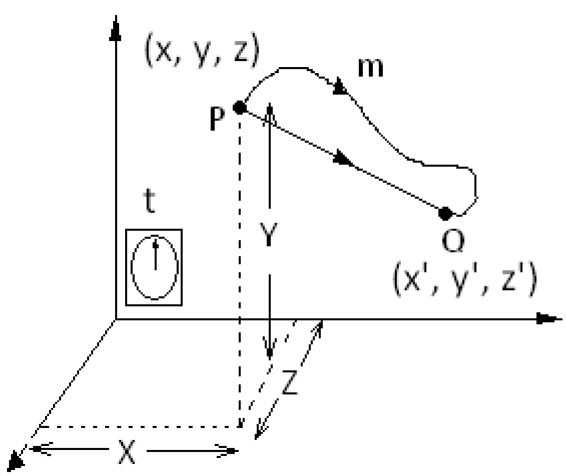
(ii) Motion in two dimensions
In this type, the motion is represented by any two of the three coordinates. Example: a body moving in a plane.

(iii) Motion in three dimensions
Motion of a body is said to be three dimensional, if all the three coordinates of the position of the body change with respect to time.
Examples : motion of a flying bird, motion of a kite in the sky, motion of a molecule, Etc

Position, Path-length and Displacement

POSITION

Choose a rectangular coordinate system consisting of three mutually perpendicular axes, labeled X-, Y-, and Z-axes. The point of intersection of these three axes is called origin (O) and serves as the reference point, the coordinates (x,y,x) of a particle at point P describe the position of the object with respect to this frame of reference. To measure the time we put clock in this system



If all the coordinate of particle remains unchanged with time then particle is considered at rest with respect to this frame of reference. If position of particle at point P given by coordinates (x, y, z) at time t and particles position coordinates are (x', y', z') at time t' , that is at least one coordinates of the particle is changed with time then particle is said to be in motion with respect to this frame of reference

PATH LENGTH

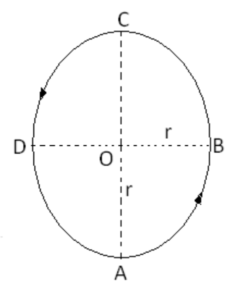
The path length of an object in motion in a given time is the length of actual path traversed by the object in the given time. As shown in figure actual path travelled by the particle is PmO . Path length is always positive

DISPLACEMENT

The displacement of an object in motion in a given time is defined as the change in a position of the object, i.e., the difference between the final and initial positions of the object in a given time. It is the shortest distance between the two positions of the object and its directions is from initial to final position of the object, during the given interval of time. It is represented by the vector drawn from the initial position to its final position. As shown in figure. Since displacement is vector it may be zero, or negative also

Solved numerical

Q) A particle moves along a circle of radius r. It starts from A and moves in anticlockwise direction as shown in figure. Calculate the distance travelled by the particle and magnitude of displacement from each of following cases (i) from A to B (ii) from A to C (iii) from A to D (iv) one complete revolution of the Particle



Solution

(i)Distance travelled by particle from A to B is One fourth of circumference thus

$$path\ length = 2\pi r/4 = \pi r/2$$

Displacement $|AB| = \sqrt{(OA)^2 + (OB)^2} = \sqrt{r^2 + r^2} = \sqrt{2}\, r$

(ii) Distance travelled by the particle from A to C is half of the circumference

$$path\ length = 2\pi r/2 = \pi r$$

Displacement

$$|AC| = r+r = 2r$$

(iii) Distance travelled by the particle from A to D is three fourth of the circumference

$path\ length = 2\pi r\ 3/4 = 3/2 \pi r$

Displacement AD
$|AD| = \sqrt{(OA)^2 + (OD)^2} = \sqrt{r^2 + r^2} = \sqrt{2}\, r$

(iv) For one complete revolution total distance is equal to circumference of circle

$$\text{Path length} = 2\pi r$$

Since initial position and final position is same displacement is zero

Speed and velocity

Speed

It is the distance travelled in unit time. It is a scalar quantity.

$$speed = path\ length/time$$

Solved numerical

Q) A motorcyclist covers 1/3rd of a given distance with speed 10 kmh^{-1}, the next 1/3rd at 20 kmh^{-1} and the last 1/3rd at of 30kmh^{-1}. What is the average speed of the motorcycle for the entire journey

Solution:

Let total distance or path length be 3x Time taken for first 1/3rd path length

$$t_1 = path\ length/speed = x/10\ hr$$

Time taken for second 1/3rd path length

$$t_1 = path\ length/speed = x/20hr$$

Time taken for third 1/3rd path length

$$t_1 = patn\ length/speed = x/30\ hr$$

Total time taken to travel path length of 3x is, $t = t_1+t_2+t_3$
Substituting values of t_1 t_2 and t_3 in above equation we get

$$t = x/10 + x/20 + x/30 = 11x/\ 60\ \ hr$$

Form the formula for speed

$$speed = path\ length/time$$

$$speed = path\ length/time$$

$$speed = 3x/11x/60 = 180/11 = 16.36\ kmh^{-1}$$

Velocity

The velocity of a particle is defined as the rate of change of displacement of the particle. It is also defined as the speed of the particle in a given direction. The velocity is a vector quantity. It has both magnitude and direction.

$$velocity = displacement/time$$

Units for velocity and speed is m s^{-1} and its dimensional formula is LT^{-1}.

Uniform velocity

A particle is said to move with uniform velocity if it moves along a fixed direction and covers equal displacements in equal intervals of time, however small these intervals of time maybe.

Non uniform or variable velocity

The velocity is variable (non-uniform), if it covers unequal displacements in equal intervals of time or if the direction of motion changes or if both the rate of motion and the direction change.

Average velocity

Let s_1 be the position of a body in time t_1 and s_2 be its position in time t_2 The average velocity during the time interval $(t_2 – t_1)$ is defined as

$$v = s_2 - s_1/t_2 - t_1 = \Delta s/\Delta t$$

Average speed of an object can be zero ,positive or zero. It depends on sign of displacement. In general average speed of an object can be equal to or greater than the magnitude of the average velocity

Instantaneous velocity

It is the velocity at any given instant of time or at any given point of its path. The instantaneous velocity v is given by

$$v = \lim_{\Delta t \to 0} \Delta s/\Delta t = ds/dt$$

Acceleration

If the magnitude or the direction or both of the velocity changes with respect to time, the particle is said to be under acceleration. Acceleration of a particle is defined as the rate of change of velocity. If object is performing circular motion with constant speed then also it is accelerated motion as direction of velocity is changing

Acceleration is a vector quantity.

$$acceleration = change\ in\ velocity/time$$

If u is the initial velocity and v, the final velocity of the particle after a time t, then the acceleration,

$$a = v - u/t$$

Its unit is m s−2 and its dimensional formula is LT^{-2}

The instantaneous acceleration is

$$a = dv/dt = d/(ds/dt) = d^2s/dt^2$$

If the velocity decreases with time, the acceleration is negative. The negative acceleration is called retardation or deceleration

Equations of motion

Motion in straight line with uniform velocity

If motion takes place with uniform velocity v on straight line the

Displacement in time t , S = vt - - - -eq(1)

Acceleration of particle is zero

Motion in a straight line with uniform acceleration – equations of motion

Let particle moving in a straight line with velocity u (velocity at time t = 00 and with uniform acceleration a. Let its velocity be v at the end of the interval of time t (final velocity at time t). Let S be the displacement at the instant t acceleration a is

$$a = v - u/t \ \ or$$

$$v = u + at - - - (2)$$

If u and a are in same direction 'a" is positive and hence final velocity v will be more than initial velocity u, velocity increases If u and a are in opposite direction final velocity v will be less than initial velocity u. Velocity is decreasing. And acceleration is negative

Displacement during time interval t = average velocity ×t

$$S = v + u/2 \times t \quad - -(3)$$

Eliminating v from equation 3 and equation 2 we get

$$S = u + at + u/2 \times tS = ut + 1/2 \quad at2 - - - (4)$$

Another equation can be obtained by eliminating t from equation 2 and equation3

$$v = u + at$$

$$t = v - u/a$$

$$S = v + u/2 \times v - u/a$$

$$S = v^2 - u^2 /2a$$

$$v^2 = u^2 + 2aS - - - (5)$$

Distance transverse by the particle in n^{th} second of its motion The velocity at the beginning of the nth second = u +a (n-1)
The velocity at the end of n^{th} second = u+an
Average velocity during n^{th} second v_{ave}

$$v_{ave} = u + a\ (n - 1) + u + an/2$$

$$v_{ave} = u + 1/2\ \ (2n - 1)$$

Distance during this one second
Sn = average velocity × time

$$S_n = u + 1/2\ \ (2n - 1) \times 1$$

$$S_n = u + 1/2\ \ (2n - 1) - - - eq(6)$$

The six equations derived above are very important and are very useful in solving problems in straight-line motion Calculus method of deriving equation of motion
The acceleration of a body is defined as

$$a = dv/dt$$

$$dv = adt$$

Integrating we get v = at+A

Where A is constant of integration . For initial condition t = 0, v = u (initial velocity)
we get A = u
∴ v = u +at
We know that instantaneous velocity v

$$v = ds/dt$$

ds = adt
displacement ds = vdt = (u+at)dt
integrating above equation

$$S = ut + 1/2\ at^2 + B$$

B is integration constant
At t = 0, S = 0 yields B = 0

$$\therefore S = ut + 1/2\ at^2$$

Acceleration a

$$a = dv/dt = dv/dS \cdot dS/dt = v\ dv/dS$$

$$\therefore a = v\ \ dv\ /dS\ adS = v \cdot dv$$

Integrating we get

$$aS = v^2/2 + C$$

Where C is integration constant
Applying initial condition , where S = 0, v = u we get

$$0 = u^2/2 + C$$

$$Or\ C = -u^2/2$$

$$\therefore aS = v^2/2 - u^2/2$$

$$v^2 = u^2 + 2aS$$

If S_1 and S_2 are the distances traversed during n seconds and (n-1) seconds

$$S_1 = un + 1/2\ an^2$$

$$S_2 = (n-1) + 1/2\ (n-1)^2$$

Displacement in n^th^ second

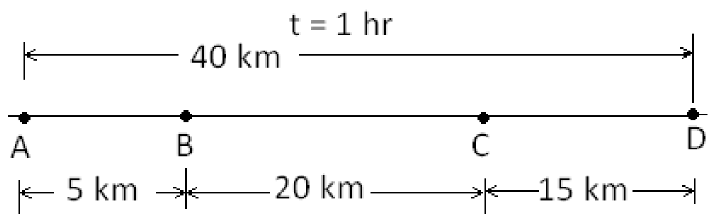
$$S_n = S_1 - S_2$$

$$S_n = un + 1/2an^2 - (n-1) - 1/2\ (n-1)^2$$

$$S_n = u + 1/2\ (2n-1)$$

Solved numerical

Q) The distance between two stations is 40 km. A train takes 1 hour to travel this distance. The train, after starting from the first station, moves with constant acceleration for 5km, then it moves with constant velocity for 20 km and finally its velocity keeps on decreasing continuously for 15 km and it stops at the other station.
Find the maximum velocity of the train.



Motion is divided in three parts Motion between point A and B is with constant acceleration Here initial velocity u = 0 and final velocity at point B = vmax Let time interval be t_1 From equation

$$S = v + u/2 \times t$$

$$5 = v_{max} + 0/2 \times t_1$$

$$t_1 = 10/v_{max}$$

Motion between point B and C is with constant velocity v_{max}
Let time period t_2
Form formula S = vt

$20 = v_{max}\ t_2$

$$t_2 = 20/v_{max}$$

Motion between point C and D is with retardation
Initial velocity is v_{max} and final velocity v= 0 let time interval t_3
From formula

$$S = v + u/2 \times t$$

$$15 = 0 + v_{max}/2 \times t_3$$

$$t_3 = 30/v_{max}$$

Total time taken is 1 hr

$T = t_1 + t_2 + t_3$

$$1 = 10/v_{max} + 20/v_{max} + 30/v_{max}$$

$\therefore V_{max}$=60 km h^{-1}

Q) A certain automobile manufacturer claims that its sports car will accelerate from rest to a speed of 42.0 m/s in 8.0 s. under the important assumption that the acceleration is constant

(i) Determine the acceleration
(ii) Find the distance the car travels in 8s
(iii) Find the distance travelled in 8th s

Solution

(a) Here initial velocity u = 0 and final velocity v = 42 m/s

From formula

$$a = v - u\,/t$$

$$a = 42 - 0\,/8 = 5.25\ ms^{-2}$$

(b) Distance travelled in 8.0s

From formula

$$S = ut + 1/2\ at^2$$

$$S = (0)(t) + 1/2\ (5.25)(8)^2 = 168\ m$$

(c) distance travelled in 8th second.

From formula

$$S_n = u + 1/2\ (2n - 1)$$

$$S_n = 0 + 1/2\ (5.25)(2 \times 8 - 1) = 39.375\ m$$

Q) Motion of a body along a straight line is described by the equation x = t_3 + $4t_2$ – 2t +5 where x is in meter and t in seconds

(a) Find the velocity and acceleration of the body at t = 4s

(b) Find the average velocity and average acceleration during the time interval

from t = 0 to t = 4 s

Solution

(a)We have to find instantaneous velocity at t = 4s

$$v = dx/dt = d/dt\ (t^3 + 4t^2 - 2t + 5)$$

$$v = d/dt\ t^3 + 4d/dt\ t^2 - 2d/dt\ t + d/dt\ 5$$

$$v = 3t^2 + 4 \times 2t - 2$$

$$v = 3t^2 + 8t - 2$$

Thus we get equation for velocity, by substituting t = 4 in above equation we get instantaneous velocity at t = 4

$v=3(4)^2 + 8(4) - 2$

v = 78 m/s

To find instantaneous acceleration at t = 4s

$$a = dv/dt = d/dt\ (3t^2 + 8t - 2)$$

a =6t +8

Thus we get equation for acceleration, by substituting t=4 in equation for acceleration we get instantaneous acceleration t=4

a = 6(4) + 8

a=32 m s^{-2}

(b)Average velocity

Final position of object at time t = 4 s

$X4 = (4)^3 + 4(4)^2 - 2(4) + 5 = 125$

Initial position of object at time t = 0 s

$X0 = (0)^3 + 4(0)^2 - 2(0) + 5 = 5$

Displacement = 125 – 5 = 120 m, time interval t =4 seconds Average velocity = Displacement / time = 120/4 = 30 ms^{-1}

Average acceleration Initial velocity t = 0 from equation for velocity

$$v = 3t2 + 8t - 2$$

$$v = 3(0)2 + 8(0) - 2 = -2\ ms^{-1}$$

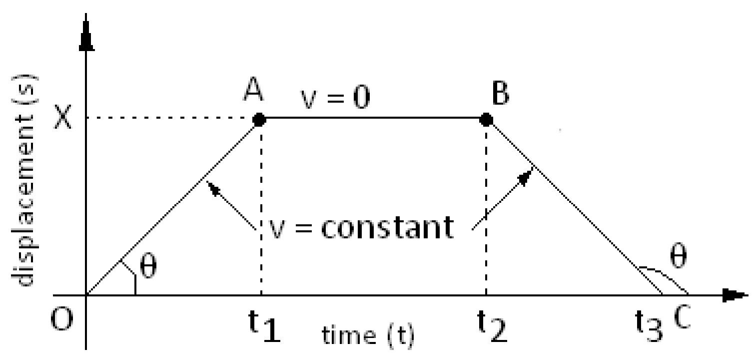
∴ Initial velocity u = -2 ms^{-1}
Final velocity is calculated as 78 ms^{-1}
From formula for average acceleration

$$a = v - u/t = 78 - (-2)/4 = 20ms^{-2}$$

Graphical representation of motion

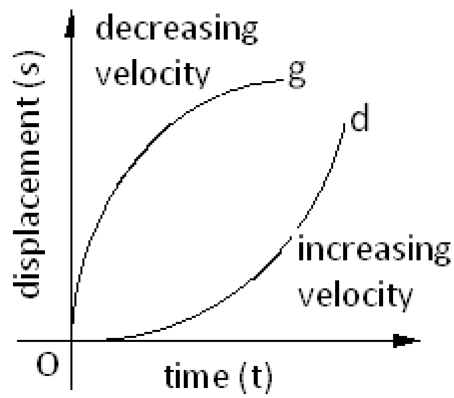
(1) Displacement – time graph:

If displacement of a body is plotted on Y-axis and time on X-axis, the curve obtained is called displacement-time graph. The instantaneous velocity at any given instant can be obtained from the graph by finding the slop of the tangent at the point corresponding to the time



graph(A)

In graph(A) object started to move with constant velocity (a = 0) at time t = 0 from origin. Object is going away represented by OA, at time t_1 object reach position X ,note slope of graph AO is positive and constant . For time period t_1 to t_2 object have not changed its position thus velocity is zero. Slope of graph is zero For time period t_2 to t_3 object started to move towards its original position at time t_2 and reaches original position at time t_3 . Here velocity is constant (a=0) as slope of graph is constant. And reaching original position as slope is negative

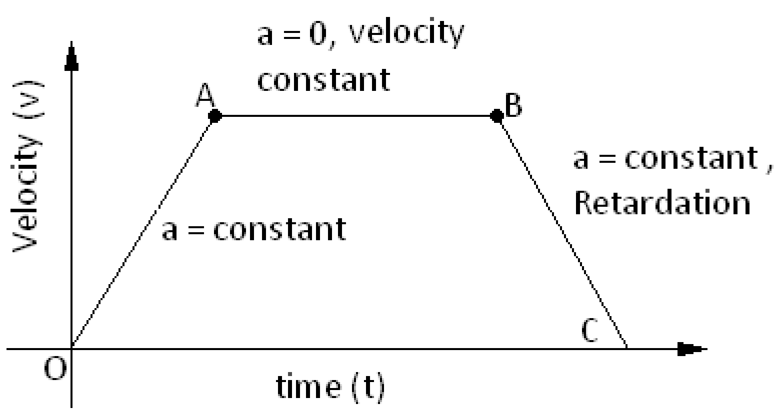


graph (B)

In graph(B) motion represented by Og is decelerated motion as slope is decreasing with time, hence velocity is decreasing. However object is moving away from origin Motion represented by Od is accelerated as slope is continuously increasing with time, it indicates that velocity is increasing or acceleration is positive , object is moving away from origin

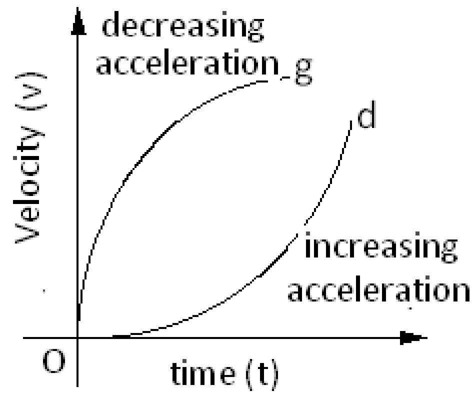
(2) Velocity-time graph

If Velocity of a body is plotted on Y-axis and time on X-axis, the curve obtained is called velocity-time graph. The instantaneous acceleration at any given instant can be obtained from the graph by finding the slop of the tangent at the point corresponding to the time

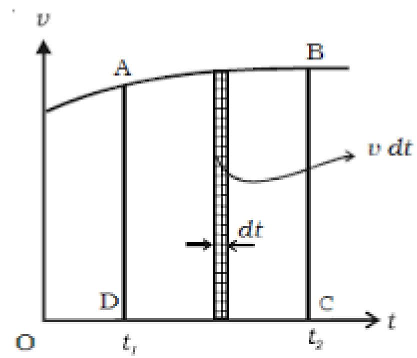


graph (C)

Graph AB is parallel straight indicate object is moving with constant velocity or acceleration is zero Graph OA is oblique straight line slope is positive indicate object is uniformly accelerated Graph BC is oblique straight line slope is negative indicated object is uniformly Decelerated



Graph Og represents decreasing acceleration as slop is decreasing with time Graph Od represent increasing acceleration as slop is increasing with time When the velocity of the particle is plotted as a function of time, it is velocity-time graph. Area under the curve gives displacement



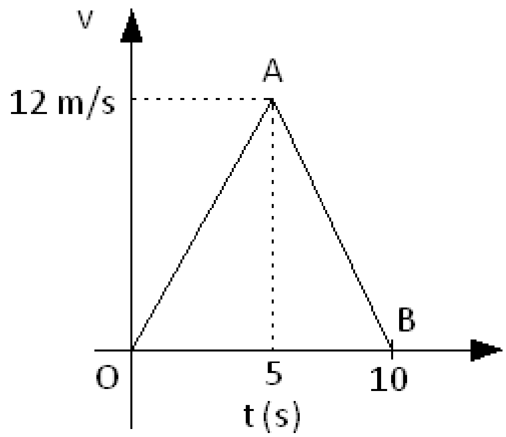
We know that

$$v = dS/dt$$

$$dS = v \cdot dt$$

Solved numerical

Q) The v – t graph of a particle moving in straight line is shown in figure. Obtain the distance travelled by the particle from (a) t = 0 to t = 10s and from (b) t = 2s to 6s



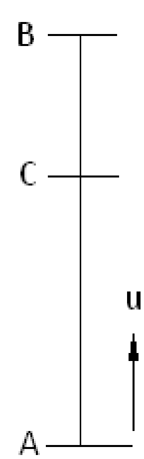
Solution:

(a) Distance travelled in time period t = 0 to t = 10s is area of triangle OAB = (1/2)×10×12 = 60 m
(b) Distance in time period t = 2 to t = 6s

From graph slope of line OA is 2.4 m/s^2 Initial velocity at t = 2 sec u = 4.8 thus using formula
$X = ut + (1/2)at^2$ here time period is 3 sec
$X_1 = (4.8)(3) + (1/2)(2.4)(3)^2 = 25.2$
For segment A to B acceleration is 2.4 time period 1s u = 5
$X_2 = (12)(1) - (1/2)(2.4)(1)^2 = 10.8$
Thus distance = 25.2 +10.8 = 36 m
Vertical motion under gravity When an object is thrown vertically upward or dropped from height, it moves in a vertical straight line. If the air resistance offered by air to the motion of the object is neglected, all objects moving freely under gravity will be acted upon by its weight only This causes vertical acceleration g having value 9.8 m/s2, so the equation for motion in a straight line with constant acceleration can be used. In some problems it is convenient to take the downward direction of acceleration as positive, in such case if the object is moving upward initial velocity should be taken as negative and displacement positive. If object is moving downwards then, initial velocity should be taken as positive and displacement negative.

Projection of a body vertically upwards

Suppose an object is projected upwards from point A with velocity u If we take down word direction of g as Negative then



(i) At a time t its velocity $v = u - gt$
(ii) At a time t, its displacement from A is gen by
$S = ut - (1/2)gt^2$
(iii) Its velocity when its displacement S is given by
$v^2 = u^2 - 2gS$

(iv) When it reaches the maximum height, its velocity v = 0.
This happens when t = u/g. The body is instantaneously rest From formula

$$V = u - gt$$

$$t = v/g$$

(v) The maximum height reached. At maximum height final velocity v = 0 and S = H thus
From equation

$$v^2 = u^2 - 2gS$$

$$0 = u^2 - 2gH$$

$$H = u^2/2g$$

(vi) Total time to go up and return to the point of projection
Displacement S = 0 Thus from formula

$$S = ut - (1/2)\,gt^2$$

$$0 = ut - (1/2)\,gt^2$$

$$T = 2u/g$$

(vii) At any point C between A and B, where AC = s, the velocity v is given by

$$v = \pm\sqrt{u^2 - 2gS}$$

The velocity of body while crossing C upwards =

$$v = +\sqrt{u^2 - 2gS}$$

The velocity of body while crossing C downwards

$$v = -\sqrt{u^2 - 2gS}$$

Magnitudes of velocities are same

Solved numerical

Q) A body is projected upwards with a velocity 98 m/s. Find

(a) the maximum height reached
(b) the time taken to reach maximum height
(c) its velocity at height 196 m from the point of projection
(d) velocity with which it will cross down the point of projection and
(e) the time taken to reach back the point of projection

Solution:

(a) Maximum height

$$H = u^2/2g = (98)^2/2 \times 9.8 = 490\ m$$

(c) Time taken to reach maximum height

$$T = u/g = 9.8/9.8 = 10s$$

(d) Velocity at a height of 196 m from the point of projection

$$v = \pm\sqrt{u^2 - 2gS}$$

$$v = \pm\sqrt{(98)^2 - 2(9.8)(196)} = \pm 75.91\ m/s$$

+75.91 m/s while crossing the height upward and -75.91 m/ while crossing it downwards
Magnitude is same but direction is opposite hence V = -u =-98 m/s

(e) The time taken to reach back the point of projection

$$T = 2u/g = (2\times 98)/9.8 = 20\ s$$

MOTION IN TWO DIMENSIONS

Position of points in two dimensions is represented in vector form

$$\vec{r}_1 = x_1\hat{\imath} + y_1\hat{\jmath} \text{ and } \vec{r}_2 = x_2\hat{\imath} + y_2\hat{\jmath}$$

If particle moves from r_1 position to r_2 position then
Displacement is final position – initial position

$$\vec{s} = \vec{r}_2 - \vec{r}_1$$
$$\vec{s} = (x_2\hat{\imath} + y_2\hat{\jmath}) - (x_1\hat{\imath} + y_1\hat{\jmath})$$
$$\vec{s} = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath}$$

If $x_2 - x_1 = dx$ and $y_2 - y_1 = dy$

$$\vec{s} = dx\hat{\imath} + dy\hat{\jmath}$$

Now

$$\vec{v} = d\vec{s}/dt = dx/dt\ \hat{\imath} + dy/dt\ \hat{\jmath}$$

$dx/dt = v_x$ is velocity of object along x-axis and $dy/dt = v_y$ is velocity along y-axis

$$\vec{v} = v_x\hat{\imath} + v_y j$$

Thus motion in two dimension is resultant of motion in two independent component motions taking place simultaneously in mutually perpendicular directions.
Magnitude of velocity

$$v = \sqrt{v_x^2 + v_y^2}$$

Angle between velocity and x-axis is
$\tan\theta = v_y/v_x$

$$\vec{a} = dv_x/dt\ \hat{i} + dv_y/dt\ \hat{j}$$
$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

Magnitude of acceleration

$$a = \sqrt{a_x^2 + a_y^2}$$

Equation of motion in vector form

$$\vec{s} = \vec{u}t + 1/2\vec{a}t^2$$
$$\vec{v} = \vec{u} + \vec{a}t$$
$$\vec{r} = \vec{r}_0 + \vec{u}t + 1/2\ \vec{a}t^2$$
$$average\ velocity = \vec{u} + \vec{v}/2$$
$$v^2 - u^2 = 2\vec{a} \cdot (\vec{r} - \vec{r}_0)$$

Solved Numerical

Q) A particle starts its motion at time t = 0 from the origin with velocity 10**j** m/s and moves
in X-Y plane with constant acceleration 8**i**+2**j**
(a) At what time its x-coordinate becomes 16m? And at that time what will be its y co-ordinate?
(b) What will be the speed of this particle at this time?

Solution:

From equation of motion

$$\vec{r} = \vec{r}_0 + \vec{u}t + 1/2\ \vec{a}t^2$$
$$x\hat{i} + y\hat{j} = (10\hat{j}) + 1/2\ (8\hat{i} + 2\hat{j})t^2$$
$$x\hat{i} + y\hat{j} = (4t^2\hat{j}) + (10t + t^2)\hat{j}$$

Thus

$$x = 4t^2$$

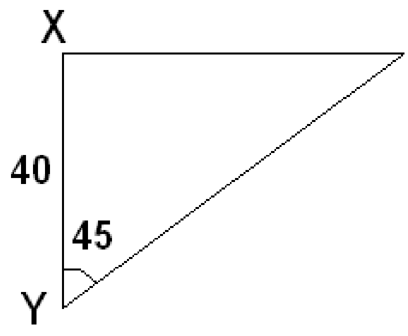
When x = 16
$16 = 4t^2$, t = 2 sec
Now $y = 10t+t^2$ at t = 2 sec
Y = 20+4 = 240 m
b)

$$\vec{v} = \vec{u} + \vec{a}t$$
$$\vec{v} = 10\hat{j} + (8\hat{i} + 2\hat{j})$$
t =2 sec
$$\vec{v} = 10\hat{j} + (8\hat{i} + 2\hat{j}) \times 2$$
$$\vec{v} = 16\hat{i} + 14\hat{j}$$
$$|\vec{v}| = \sqrt{16^2 + 14^2} = 21.26\ sec$$

Q) A bomber plane moves due east at 100km/hr over a town X at a certain time, Six

minutes later an interceptor plans sets off from station Y which is 40km due south of X and
flies north east. If both maintain their courses, find the velocity with which interceptor must fly in order to take over the bomber

Solution:



Let velocity of interceptor speed be V . Now components of velocity along North direction is Vcos45 and along East direction is Vsin45

To intercept the bomber , interceptor has to travel a distance
of 40 km in t seconds along North direction Thus 40 = Vos45t

Vt = 40√2 ---- eq(1)

Now total distance travelled by bomber = 100t + distance travelled in 6 minutes before interceptor takeoff

Distance travelled by bomber along East direction = 100t +10 km

Thus distance travelled by interceptor

100t + 10 = Vsin45t

100t +10 = Vt / √2

From eq(1)

100t +10 =40

t = 3/10 Hr

Substituting above value of t in equation (1) we get

$$t = 40\sqrt{2} \times 10/3 = 188.56\ km/hr$$

Q) A motor boat set out at 11a.m. from a position -6**i**-2**j** relative to a marker buoy and travels at steady speed of magnitude √53 on direct course to intercept a ship. This ship maintains a steady velocity vector 3**i**+4**j** and at 12 noon is at position 3**i**-4**j** from the buoy.

Find (a) the velocity vector of the motorboat (b) the time of interception and (c) the position vector of point of interception from the buoy, if distances are measured in kilometers and speeds in km/hr

Solution

Let xi+vj be the velocity vector of the motor boat

$X_2 + y_2 = 53$ ------eq(1)

Position vector of motor boat after time t = -6**i**-2**j** + (x**i**+y**j**)t

Since position of ship is given 12 noon to find position of ship at time t, time for ship = (t-1)

Position vector of ship after time t = 3**i**-**j**+(3**i**+4**j**) (t-1)

The time of interception is given by

-6**i**-2**j**+(x**i**+y**j**)t = 3**i**-**j** +(3**i**+4**j**) (t-1)

∴-6+xt = 3t

∴xt = 6 +3t -------eq(2)

And -2yt = 4t-5

∴yt = 4t-3 ------eq(3)

From equation (1), (2) and (3)

$(6+3t)^2 + (4t-3)^2 = 53t^2$

$\therefore 36+9t^2+36t+16t^2-24t+9 = 53t^2$

∴28t-12t-45 = 0

∴(2t-3) (14t +15) = 0

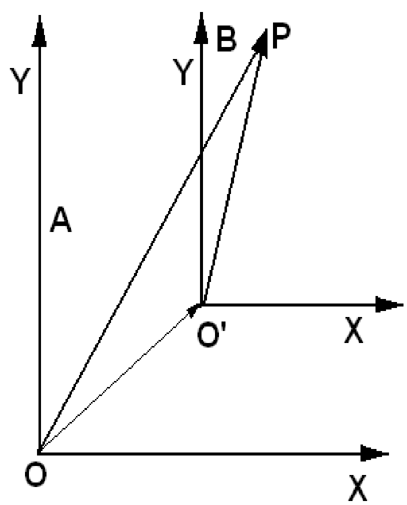
t = (3/2) hr

Time of interception = 12.30 pm

Solving equation(2) and (3) x = 7 and y = 2

Velocity vector of motor boat = 7**i**+2**j**

Point of interception = -6**i**-2**j** + (7**i** +2**j**) (3/2) = 9/2 **i** +**j**



Two frame of reference A and B as shown in figure moving
with uniform velocity with respect to each other. Such frame of references are called inertial frame of reference.

Suppose two observers, one in from A and one from B study the motion of particle P.

Let the position vectors of particle P at some instant of time with respect to the origin O of frame A be

$\vec{r}_{P,A} = \vec{OP}$

And that with respect to the origin O' of frame B be

$\vec{r}_{P,B} = \vec{OP}$

The position vector of O' w.r.t O is

$\vec{r}_{B,A} = \vec{OO'}$

From figure it is clear

$\vec{OP} = \vec{OO'} + \vec{O'P} = \vec{O'P} + \vec{OO'}$

$\therefore \vec{r}_{P,A} = \vec{r}_{P,B} + \vec{r}_{B,A}$

Differentiating above equation with respect to time we get

$$d/dt(\vec{r}_{P,A}) = d/dt\,(\vec{r}_{P,B}) + d/dt\,(\vec{r}_{B,A})$$

$$\therefore \vec{v}_{P,A} = \vec{v}_{P,B} + \vec{v}_{B,A}$$

Here $\mathbf{V}_{P,A}$ is the velocity of the particle w.r.t frame of reference A, $\mathbf{V}_{P,B}$ is the velocity of the
particle w.r.t. reference frame B and $V_{B,A}$ is the velocity of frame of reference B with respect to frame A

Suppose velocities of two particles A and B are respectively $\mathbf{V}_A$ and $\mathbf{V}_B$ relative to frame of
reference then velocity ($\mathbf{V}_{AB}$) of A with respect to B is

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

And velocity $\mathbf{V}_{BA}$ of B relative to A is

$$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

Thus

$$\vec{v}_{AB} = -\vec{v}_{BA}$$

And

$$|\vec{v}_{AB}| = |\vec{v}_{BA}|$$

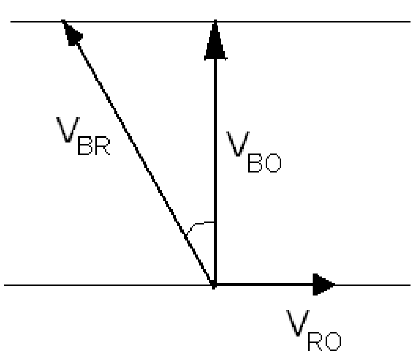
$$|\vec{v}_{AB}| = \sqrt{V_{A2} + V_{B2} - 2V_A V_B \cos\theta}$$

Also angle α made by the relative velocity with V_A is given by

$$\tan\alpha = V_B \sin\theta / V_A - V_B \cos\theta$$

Solved Numerical

Q) A boat can move in river water with speed of 8 km/h. This boat has to reach to a place from one bank of the river to a place which is in perpendicular direction on the other bank of the river. Then (i) in which direction should the boat has to be moved (ii) If the width of the river is 600m, then what will be the time taken by the boat to cross the river? The river flows with velocity 4 km/h



Suppose the river is flowing in positive X direction as shown in figure . To reach to a place in the perpendicular direction on the other bank, the boat has to move in the direction making angle θ with Y direction as shown in figure. . This angle should be such that the velocity of the boat relative to the opposite bank is in the direction perpendicular to the bank . Let V_{BR} = Velocity of boat with respect to river V_{BO} = Velocity of Boat with respect to Observer on bank V_{RO} = velocity of river with respect to observer on bank

$$\vec{V}_{BO} = \vec{V}_{BR} + \vec{V}_{RO}$$

In vector form

$$V_{BO}\,\hat{j} = -V_{BR}\sin\theta\hat{\imath} + V_{BR}\cos\theta\hat{j} + V_{RO}\,\hat{\imath}$$

$$V_{BO}\,\hat{j} = -8\sin\theta\hat{\imath} + 8\cos\theta\hat{j} + 4\hat{\imath}$$

Thus considering only x components

$$0 = -8sin\theta + 4$$

$$\Rightarrow \theta = 30^{o}$$

$$V_{BO} = 8cos\theta$$

$$V_{BO} = 8cos30$$

$$V_{BO} = 8\sqrt{3}/2 = 4\sqrt{3} = 6.93\ km/h$$

Time taken to cross river

$$t = distance/velocity = 0.6/6.93 = 0.0865\ H \quad = 5.2\ minutes$$

Projectile motion

A projectile is a particle, which is given an initial velocity, and then moves under the action of its weight alone.

When object moves at constant horizontal velocity and constant vertical downward acceleration, such a two dimension motion is called projectile.

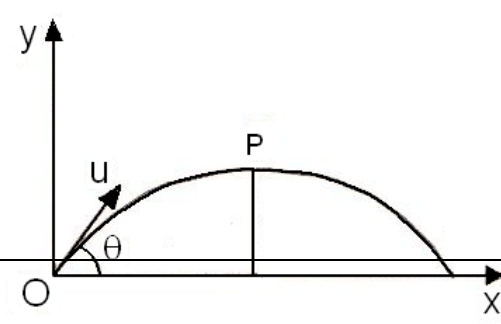
The projectile motion can be treated as the resultant motion of two independent component motion taking place simultaneously in mutually perpendicular directions. One component is along the horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to gravitational force.

Important terms used in projectile motion

When a particle is projected into air, the angle that the direction of projection makes with horizontal plane through the point of projection is called the angle of projection, the path, which the particle describes, is called the trajectory, the distance between the point of projection and the point where the path meets any plane draws through the point of projection is its range, the time that elapses in air is called as time of flight and the maximum distance above the plane during its motion is called as maximum height attained by the projectile

Analytical treatment of projectile motion

Consider a particle projected with a velocity u of an



angle θ with the horizontal earth's surface. If the earth

did not attract a particle to itself, the particle would

describe a straight line, on account of attraction of

earth, however, the particle describes a curve path

Let us take origin at the point of projection and x-axis

along the surface of earth and perpendicular to it respectively shown in figure

Here gravitational force is the force acting on the object downwards with constant

acceleration of g downwards. There if no force along horizontal direction hence

acceleration along horizontal direction is zero

Motion in x –direction

Motion in x-direction with uniform velocity

At t = 0, $X_0 = 0$ and u_x

= ucosθ

Position after time t , $x = x_0 + u_x t$

X = (ucosθ) t -----eq(1)

Velocity at t , $V_x = u_x$

V_x= ucosθ ------eq(2)

Motion in y-direction:

Motion in y-direction is motion with uniform acceleration

When t = 0, $Y_0 = 0$, u_y = usinθ and $a_y = -g$

After time 't', $V_y = y_y + a_y t$

V_y = usinθ – gt -----eq(3)

$$Y = Y_0 + u_y t + 1/2\ a_y t^2 --- e(4)$$